Lifelong Learning Programme

Národní agentura pro evropské vzdělávací programy

Reliability Aspects

Ton Vrouwenvelder
Delft University / TNO

Leonardo da Vinci
Assessment of existing structures
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EN 1990:

Reliability

- ability of a structure to fulfil all required functions during a specified period of time under given conditions

Failure probability P_f

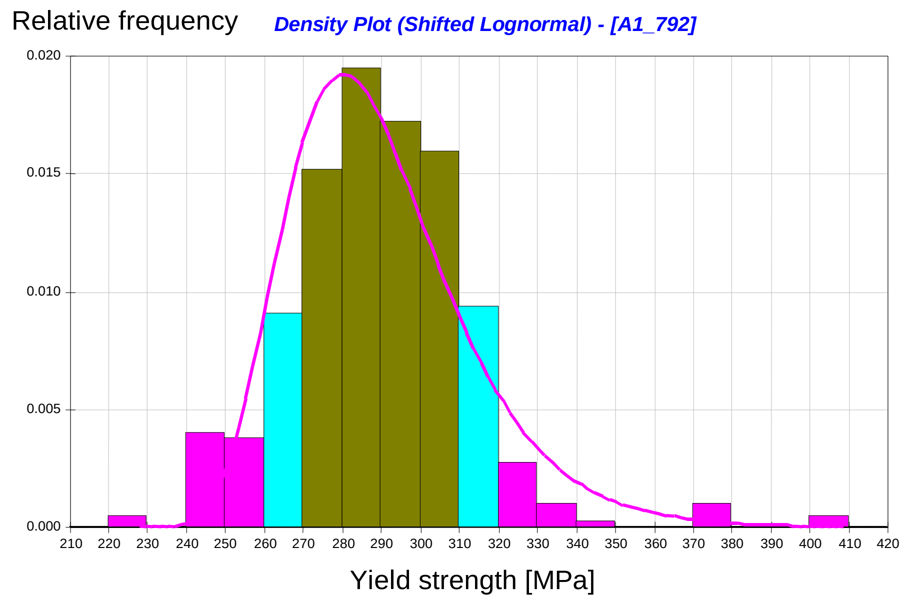
- most important measure of structural reliability

Limit State Approach

- Limit states - states beyond which the structure no longer fulfils the relevant design criteria

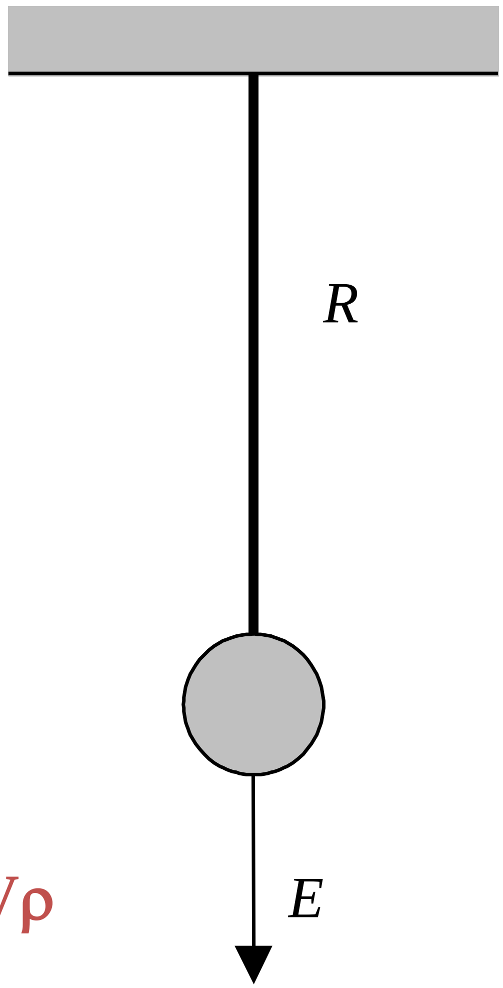
- Ultimate limit states
  - loss of equilibrium of a structure as a rigid body
  - rupture, collapse, failure
  - fatigue failure
- Serviceability limit states
  - functional ability of a structure or its part
  - users comfort
  - appearance

Uncertainties



- randomness - natural variability
- statistical uncertainties - lack of data
- model uncertainties - simplified models
- vagueness - imprecision in definitions
- gross errors - human factors
- ignorance - lack of knowledge

EXAMPLE



Resistance: $R = \pi d^2 f_y / 4$

Load effect: $E = V\rho$

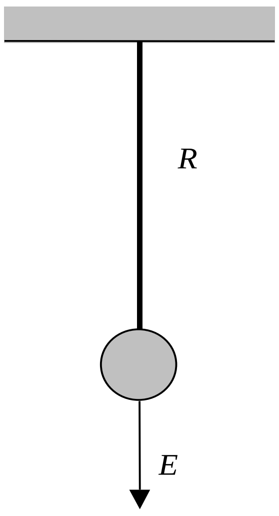
Failure if E>R or: $V\rho > \pi d^2 f_y / 4$

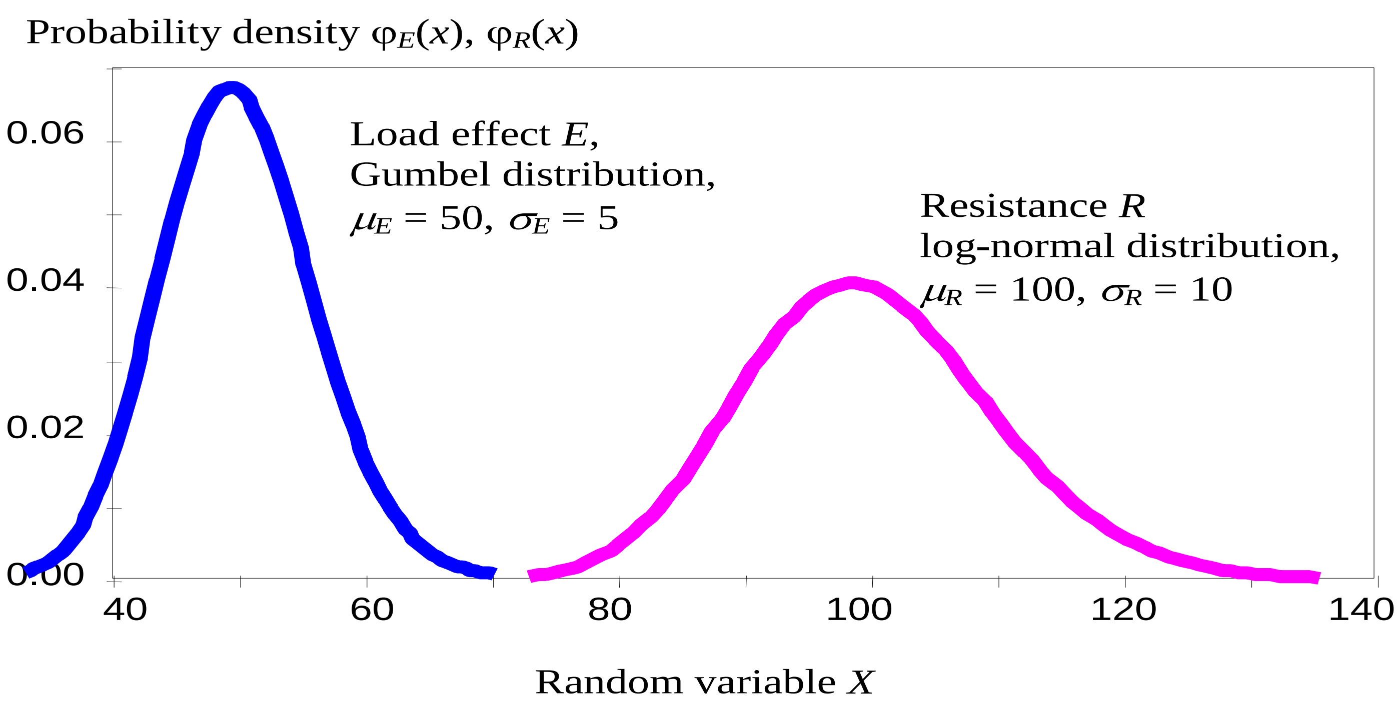
Limit state: $V\rho = \pi d^2 f_y / 4$

Limit state function: $Z = R - E = \pi d^2 f_y / 4 - V\rho$

Statistical models

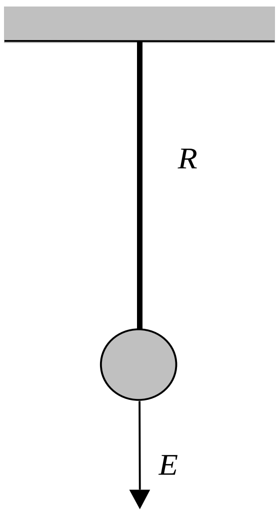
		distribution	mean	sd
R	resistance	Lognormal	100	10
E	load effect	Gumbel	50	5

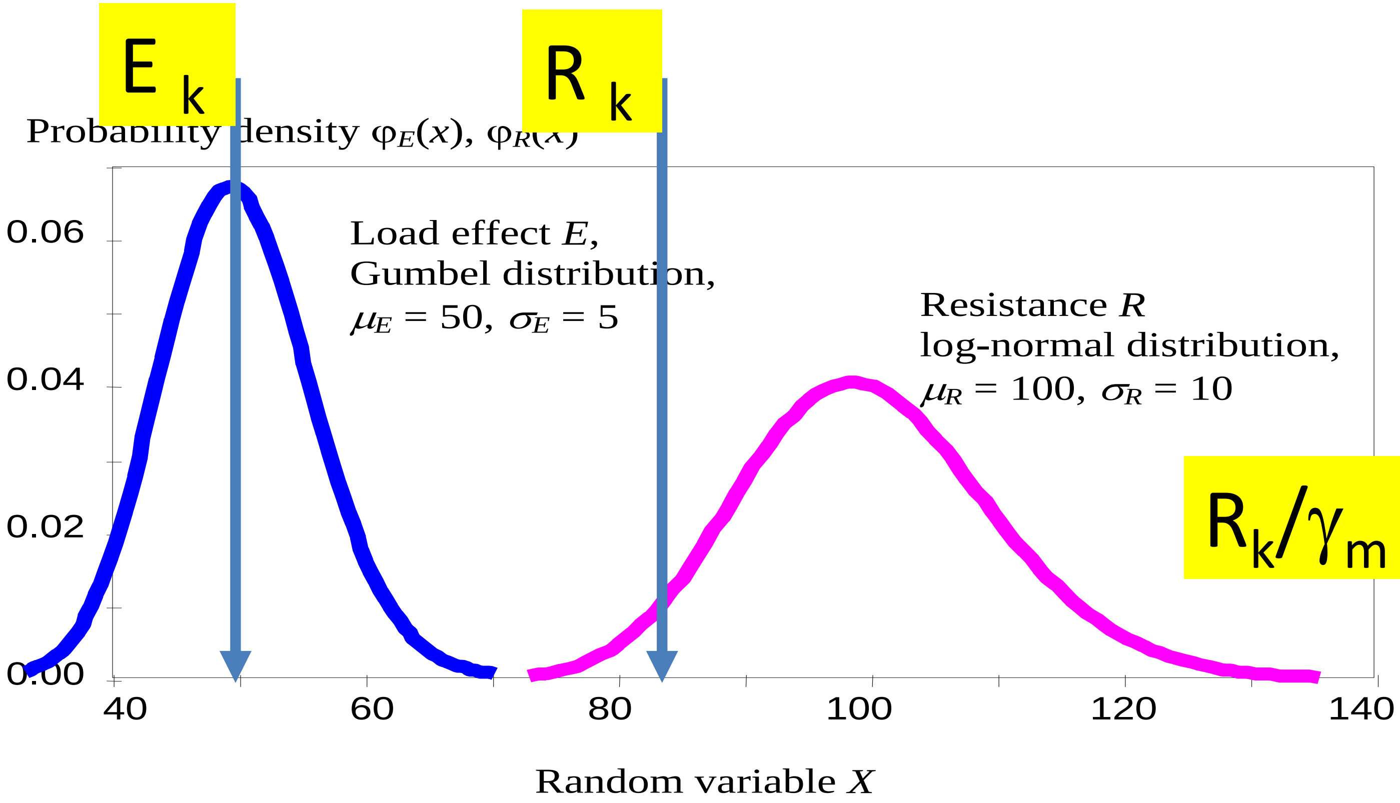




Partial factor approach

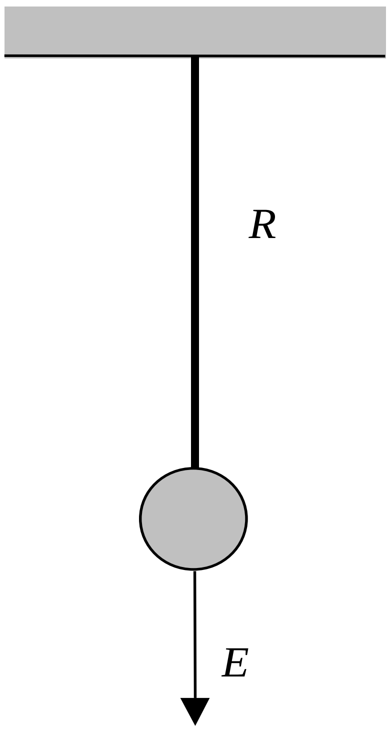
		distribution	mean	sd
R	resistance	Lognormal	100	10
E	load effect	Gumbel	50	5





$R_k/\gamma_m > E_k\,\gamma_Q$

Probabilistic approach



Z = R - E

$$P_f = P(Z < 0) = \iint_{Z(X)<0} \varphi_R(r)\varphi_E(e)\,drde$$

Techniques:

Numerical integration (NI)

Monte Carlo (MC)

First order Second moment method (FOSM)

Third moment method (accounting for skewness)

First Order Reliability Methods (FORM)

First Order Second Moment method

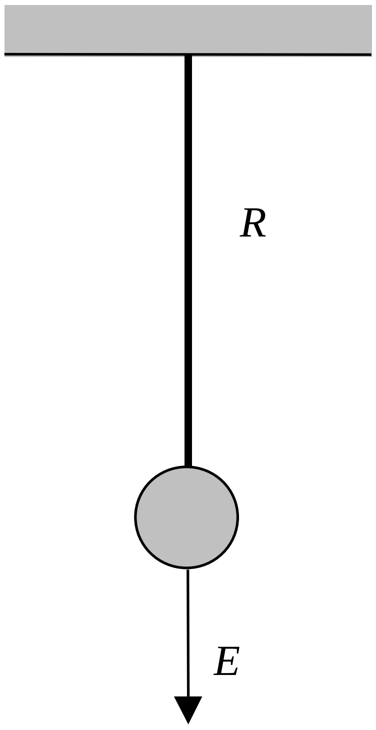
$$Z = R - E$$

$$\mu_Z = \mu_R - \mu_E = \qquad 100 - 50 = 50$$

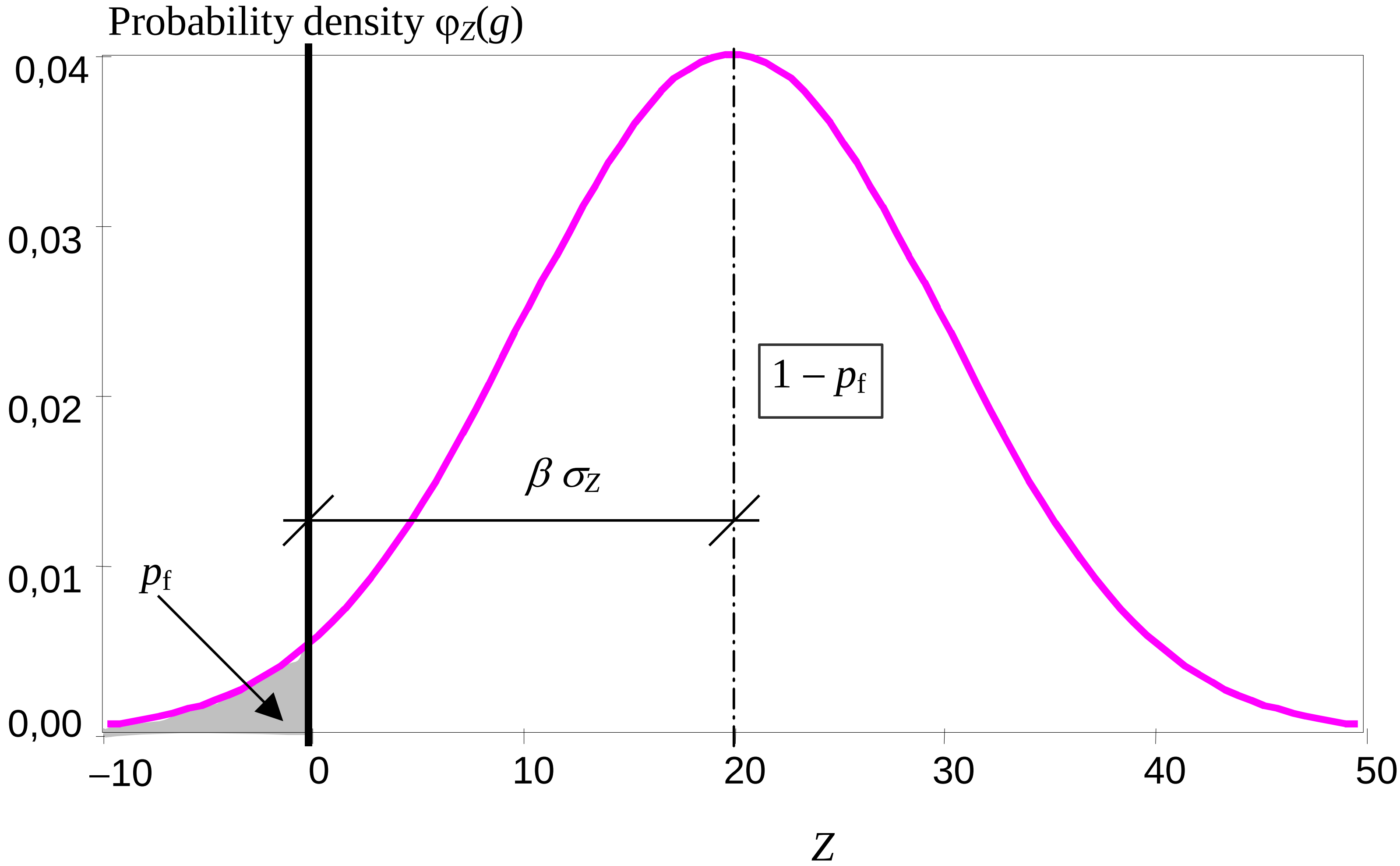
$$\sigma_Z^2 = \sigma_R^2 + \sigma_E^2 = 14^2$$

$$\beta = \mu_Z / \sigma_Z = 3.54$$

$$P_f = P(Z < 0) = \Phi_Z(0) = 0.0002$$



$\beta = \mu_Z / \sigma_Z = 3.54$ $P_f = P(Z < 0) = \Phi_Z(0) = 0.0002$



Reliability index

Probability of Failure = $\Phi(-\beta) \approx 10^{-\beta}$

β	1.3	2.3	3.1	3.7	4.2	4.7
$P(F)=\Phi(-\beta)$	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}

Relation Partial factors and beta-level:

$$\gamma = \exp\{\alpha\ \beta\ V - kV\} \approx 1 + \alpha\ \beta\ V$$

α = 0.7-0.8

β = 3.3 - 3.8 - 4,3 (life time, Annex B)

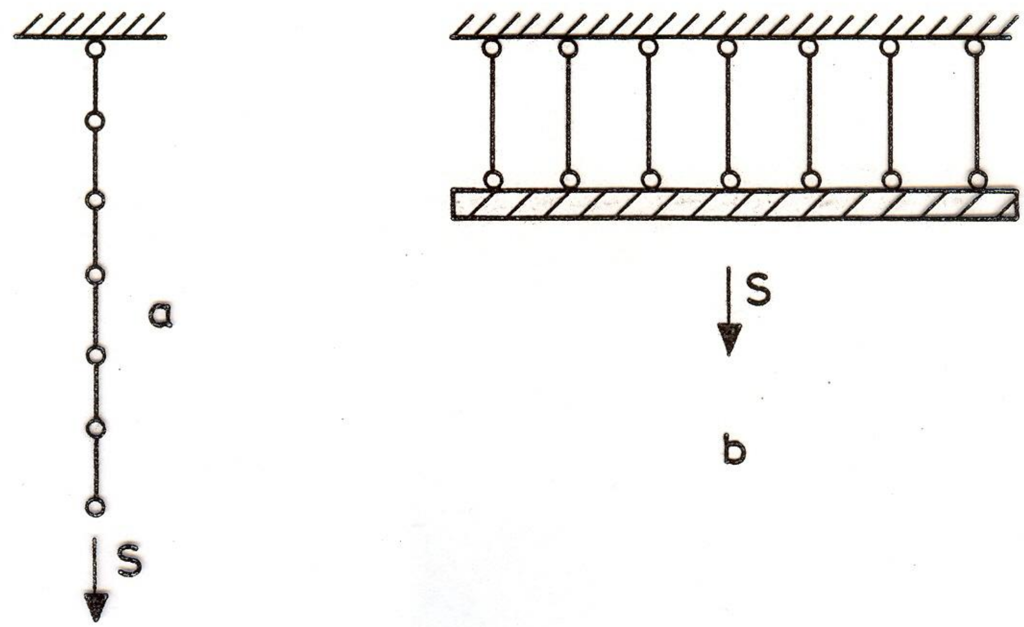
k = 1.64 (resistance)

k = 0.0 (loads)

V = coefficient of variation

Extensions

- load fluctuations
- systems
- degradation
- inspection
- risk analysis
- target reliabilities



Target levels Reliablity

Eurocode EN 1990, Annex B

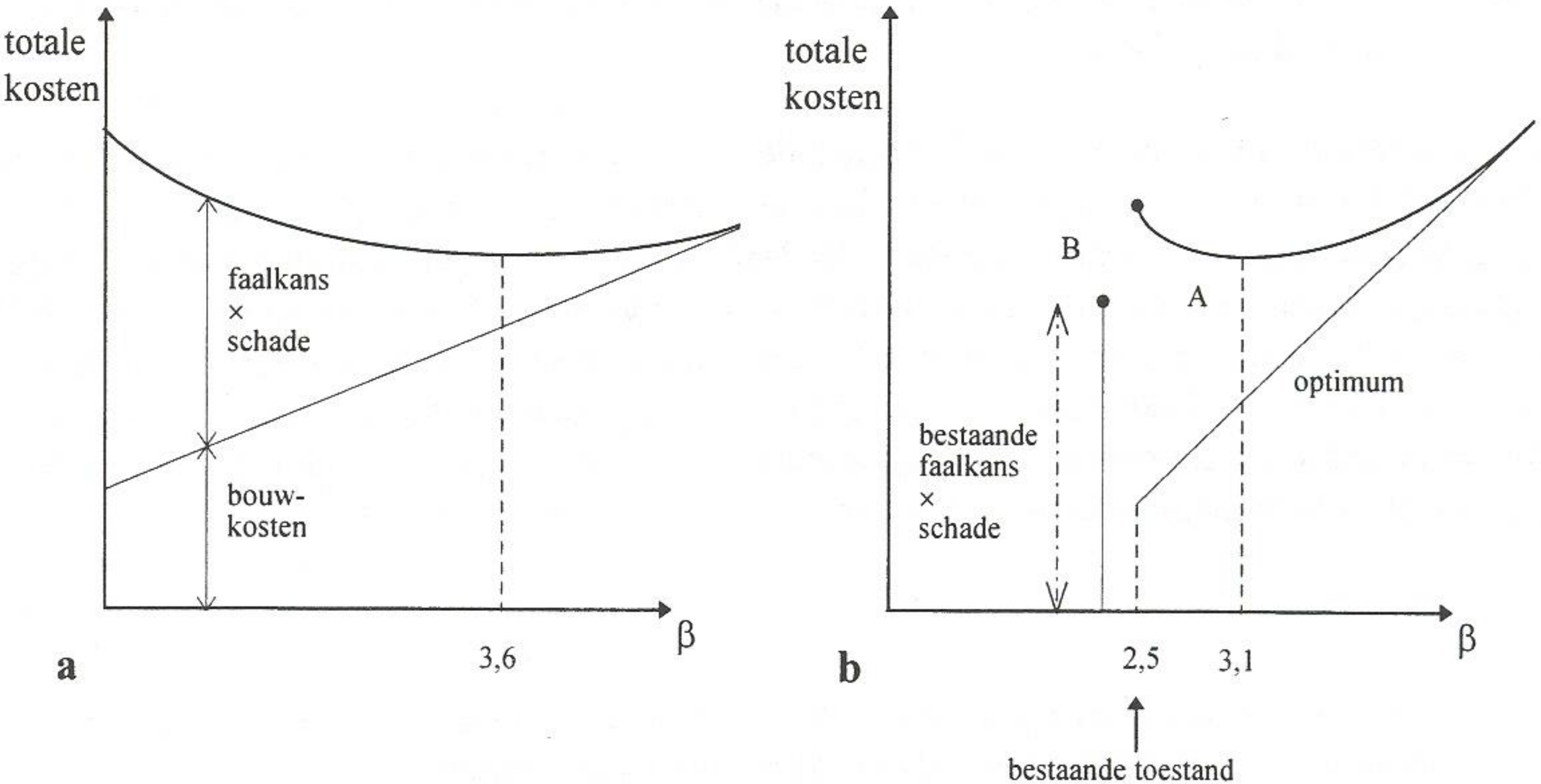
Reliability classes	Consequences for loss of human life, economical, social and environmental consequences	Reliability index β		Examples of buildings and civil engineering works
		β_a for T_a= 1 yr	β_d for T_d= 50 yr	
RC3 – high	High	5,2	4,3	Important bridges, public buildings
RC2 – normal	Medium	4,7	3,8	Residential and office buildings
RC1 – low	Low	4,2	3,3	Agricultural buildings, greenhouses

$$\gamma = \exp[(\alpha\beta - k)V] \sim 1 + \alpha\beta V$$

JCSS TARGET RELIABILITIES β
for a one year reference period

	Consequences of failure ⇒		
Cost to increase safety ⇓	Minor	Moderate	Large
Large	$\beta=3.1$ ($p_F \approx 10^{-3}$)	$\beta=3.3$ ($p_F \approx 5\ 10^{-4}$)	$\beta=3.7$ ($p_F \approx 10^{-4}$)
Normal	$\beta=3.7$ ($p_F \approx 10^{-4}$)	$\beta=4.2$ ($p_F \approx 10^{-5}$)	$\beta=4.4$ ($p_F \approx 5\ 10^{-6}$)
Small	$\beta=4.2$ ($p_F \approx 10^{-5}$)	$\beta=4.4$ ($p_F \approx 5\ 10^{-5}$)	$\beta=4.7$ ($p_F \approx 10^{-6}$)

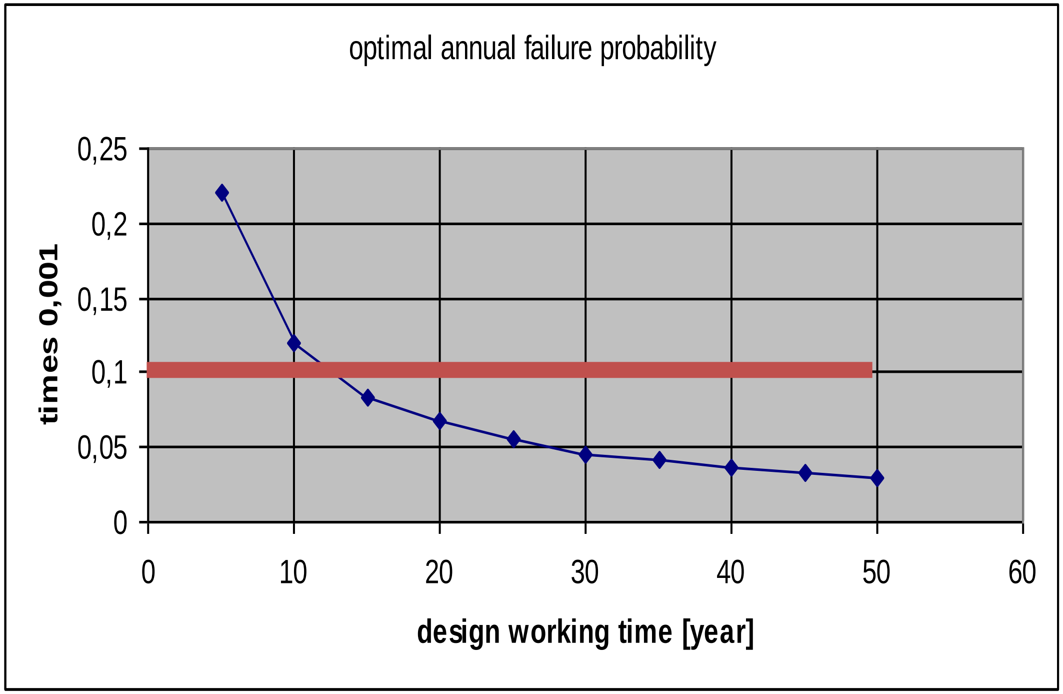
Cost optimisation / design versus assessment



$P_F = 10^{-\beta}$

Human life safety

- Include value for human life in *D*
- Still reasons for IR and SR
- Example: $p < 10^{-4}$ / year



Existing Structures (NEN 8700)

Reliability index in case of assessment

Minimum $\beta < \beta_{new} - 1.0$

Human safety: $\beta > 3.6 - 0.8 \log T$

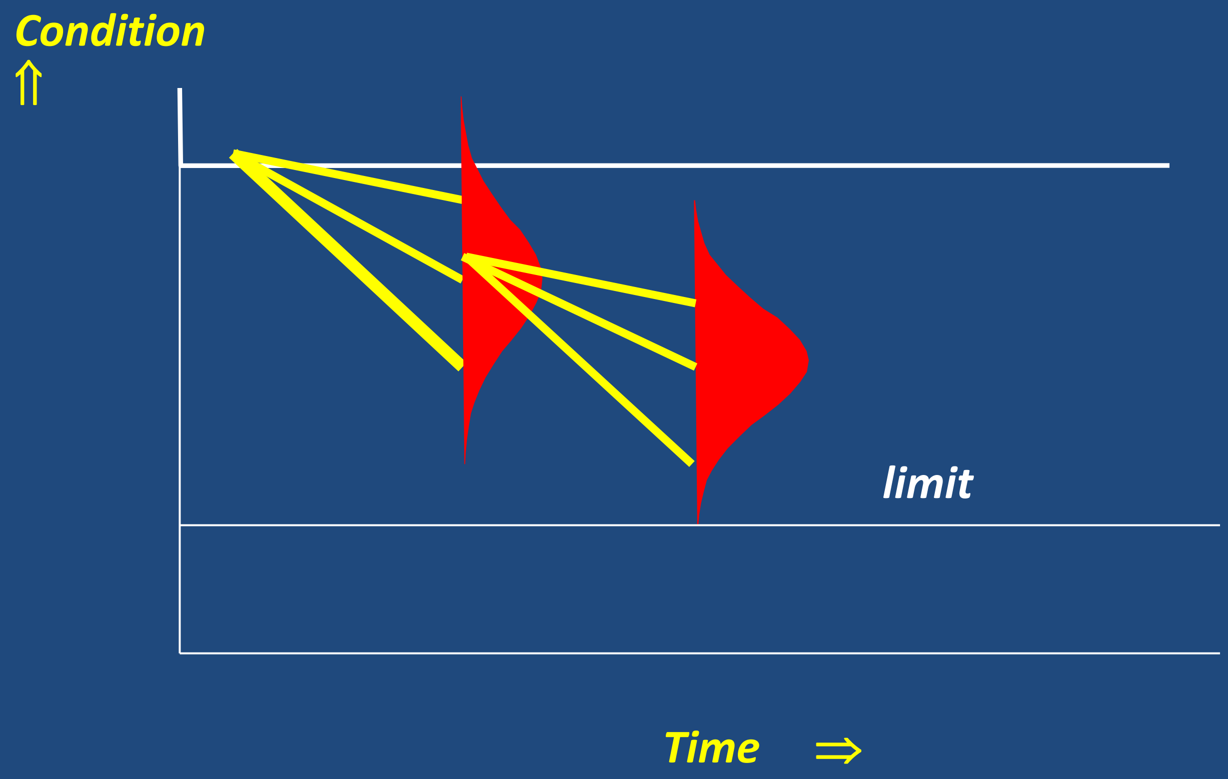
Example NEN 8700 (Netherlands)

Minimum values for the reliability index β with a minimum reference period

Consequence class	Minimum reference period for existing building	β-NEW		β-EXISTING	
		wn	wd	wn	wd
0	1 year	3.3	2,3	1.8	0.8
1	15 years	**3.3**	2,3	1.8[a]	1.1[a]
2	15 years	**3.8**	2.8	2.5[a]	2.5[a]
3	15 years	**4.3**	3.3	3.3[a]	3.3[a]

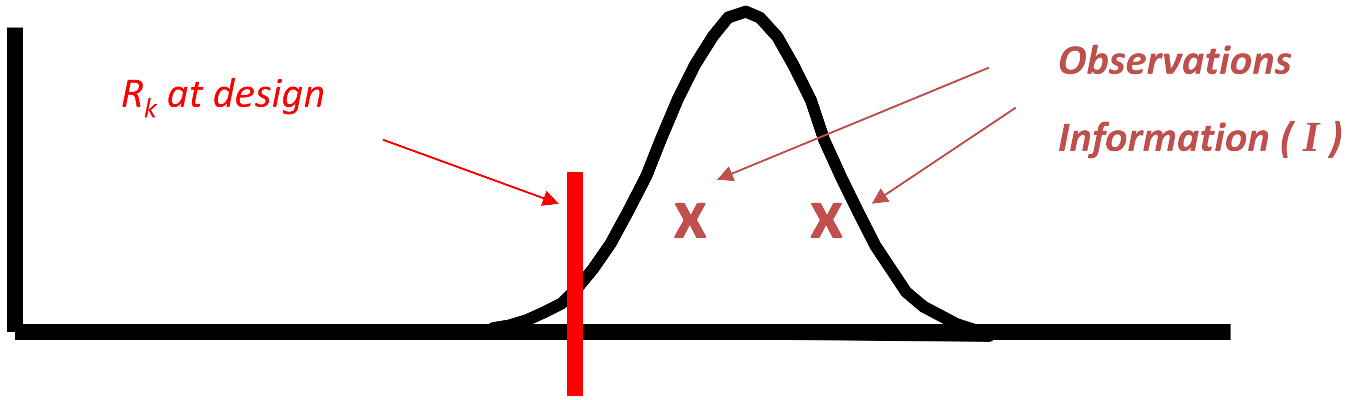
Class 0: As class 1, but no human safety involved
wn = wind not dominant
wd = wind dominant
(a) = in this case is the minimum limit for personal safety normative

Inspection en monitoring



Updating

1) Updating distributions (eg concrete strength)



2) Updating failure probability P{F | I }

Example: I = {crack = 0.6 mm}

see JCSS document on Existing Structures en ISO13822

$$\mathrm{P}(\mathrm{A} \cap \mathrm{B}) = \mathrm{P}(\mathrm{A} \mid \mathrm{B})\mathrm{P}(\mathrm{B})$$

$$\mathrm{P}(\mathrm{F} \cap \mathrm{I}) = \mathrm{P}(\mathrm{F} \mid \mathrm{I})\mathrm{P}(\mathrm{I})$$

$$P(F \mid I) = \frac{P(F \cap I)}{P(I)}$$

Two types of information I:

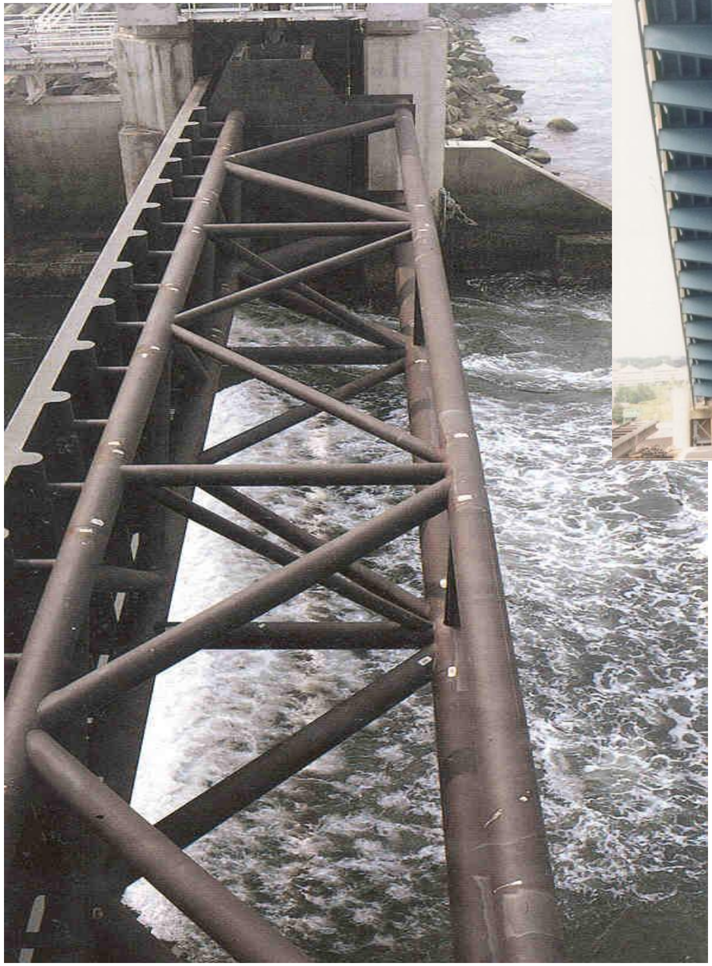
equality type: h(**x**) = 0

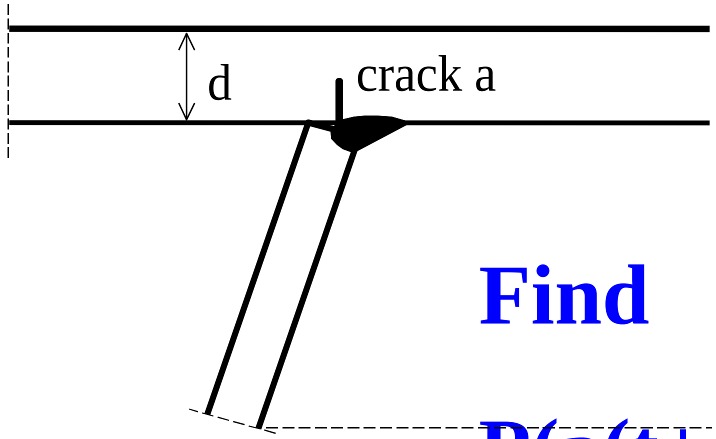
inequality type: h(**x**) < 0; h(**x**) > 0

x = vector of basic variables

$$P(F \mid I) = \frac{P(Z(t_2) < 0 \cap h(t_1) > 0)}{P(h(t_1) > 0)}$$

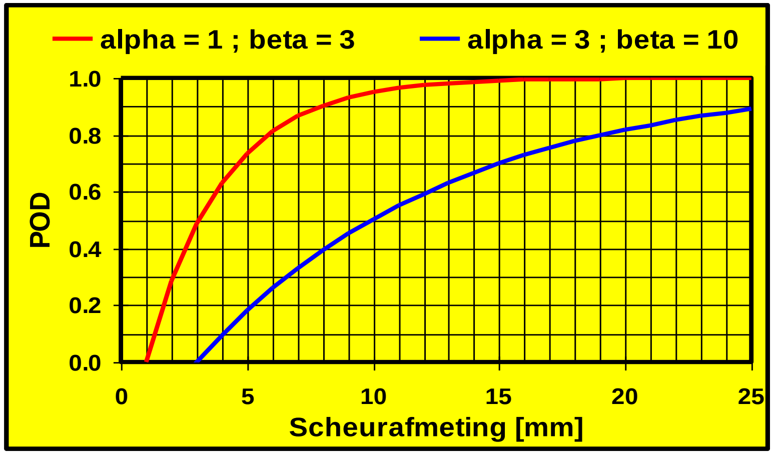
Fatigue steel structures



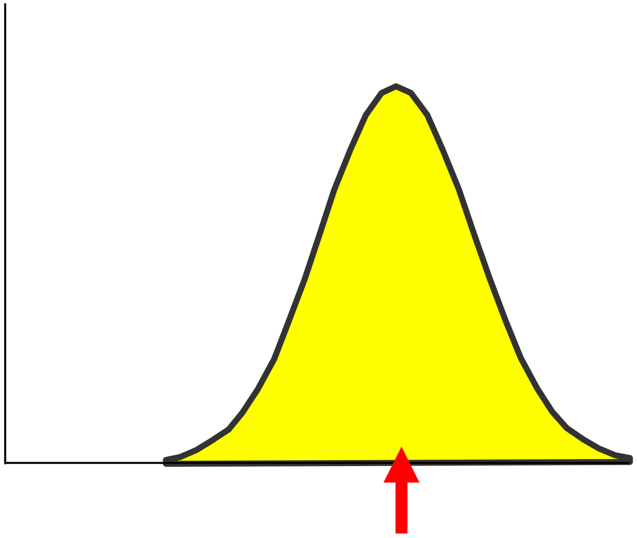


Find

$$P(a(t+\Delta t) > d \mid a(t) = ..\ \text{of}\ \ a(t) < ..)$$

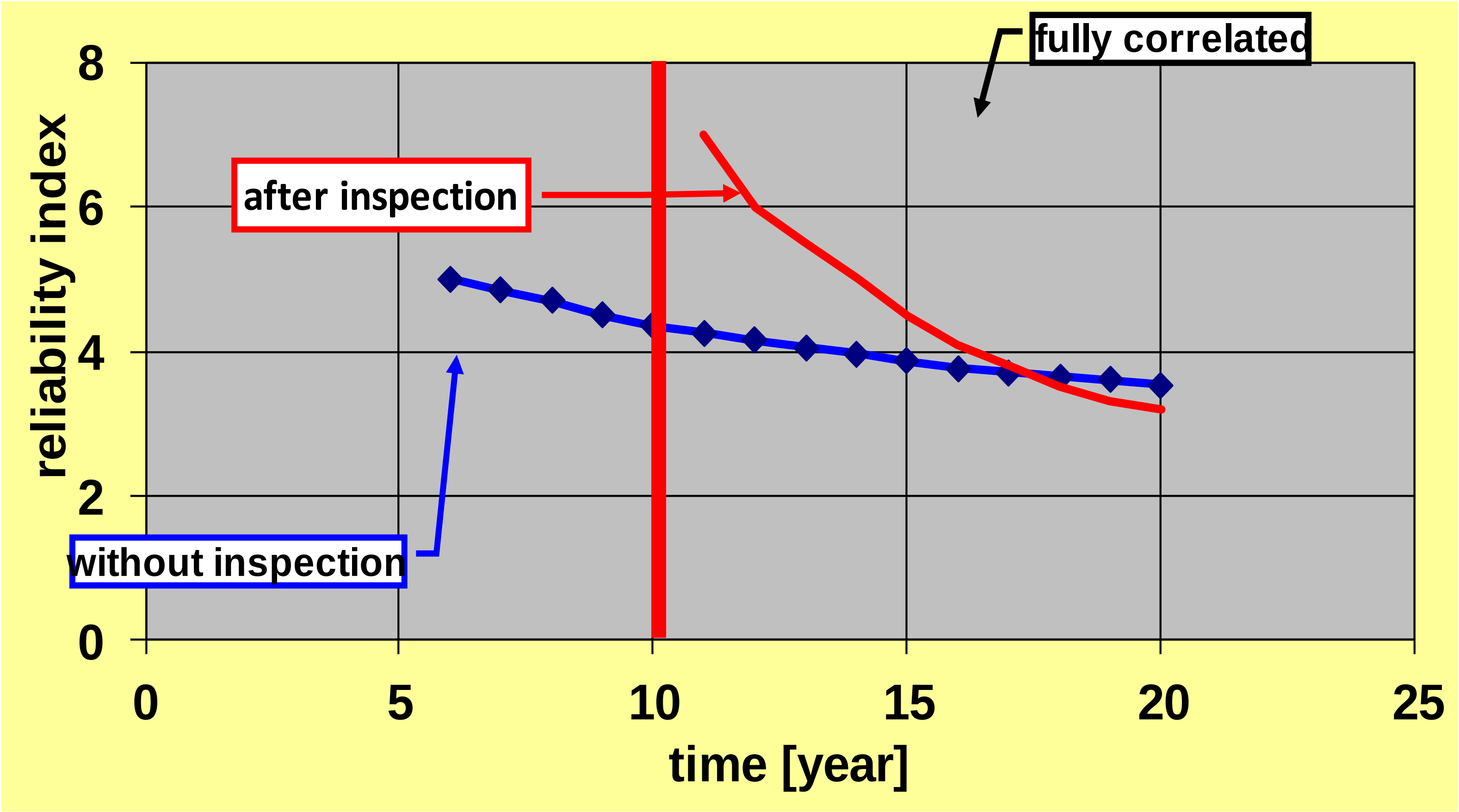


no cracks found, but?



measured 1 mm, but?

Reliabilty level Beta (one year periods)
given a crack found at t=10 a

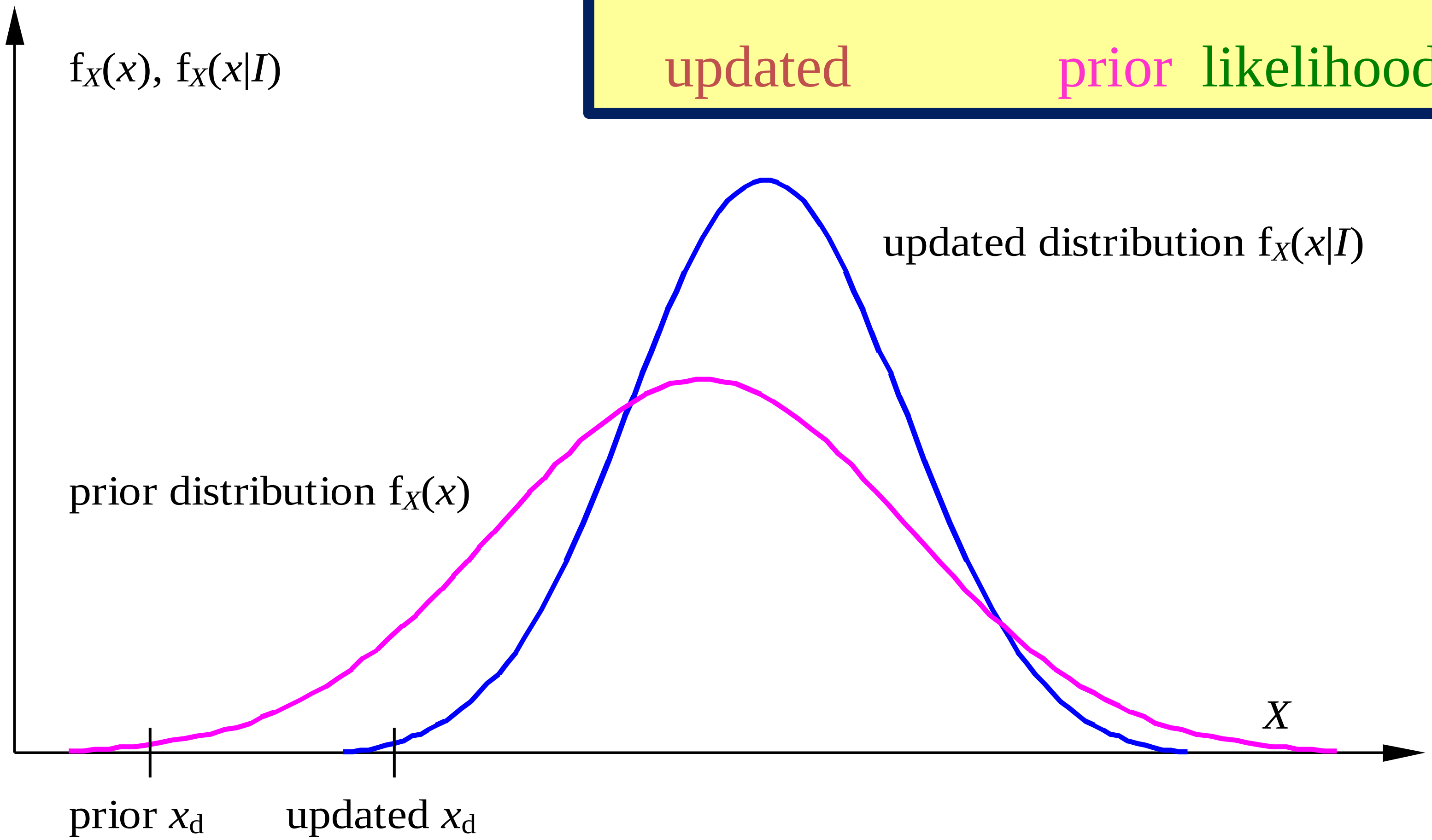


Updating distributions

$$P(x|I) = P(x)\, P(I \mid x) \,/\, P(I)$$

$$f_X(x|I) = C\; f_X(x)\, P(\, I \mid x)$$

updated prior likelihood



Formal Updating formulas

$$f_Q''(q \mid \hat{x}) = C\ f_Q'(q)\, L(\mid \hat{x} \mid q)$$

$$f_X^U(x) = \int_{-\infty}^{\infty} f_X(x|q)\, f_Q''(q \mid \hat{x})\, dq$$

Formal Updating formulas

$$f_Q^{''}(q \mid \hat{x}) = C \ f_Q^{'}(q) \, L(\mid \hat{x} \mid q)$$

$$f_X^U(x) = \int_{-\infty}^{\infty} f_X(x|q) \, f_Q^{''}(q \mid \hat{x}) dq$$

Ask the expert !

Example: Resistance with unknown mean m_R and known stand. Dev. s_R =17,5

Assume we have 3 observations with mean m_m = 350
Then m_R has s_m = 17,5/√3 = 10.
If the load is to 304 then:

m_Z= 350-304=46
s_Z =$\sqrt{(17,5^2 + 10^2)}$ = 20,2
β=2,27
P_f=0,0116

Now we have one extra observation equal to 350.
In that case the estimate of the mean m_m does not change.
The standard deviation of the mean changes to 17,5/√(4) = 8,8

m_Z= 350-304=46,
s_Z =$\sqrt{(17,5^2 + 8,8^2)}$ = 19,6,
β=2,35
P_f=0,0095

Summary Reliability aspects

- Uncertainties exist
- Probability Theory may be helpful
- Reliability targets depends on consequences of failure
- Reliability targets depend on costs of improving
- Existing structures may have a lower target reliability
- Reliability may be updated using inspection results
- There is a relation partial factor – reliability index