Affidabilità strutturale

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Assessment of existing structures
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EN 1990:

Reliability

- ability of a structure to fulfil all required functions during a specified period of time under given conditions

Failure probability P_f

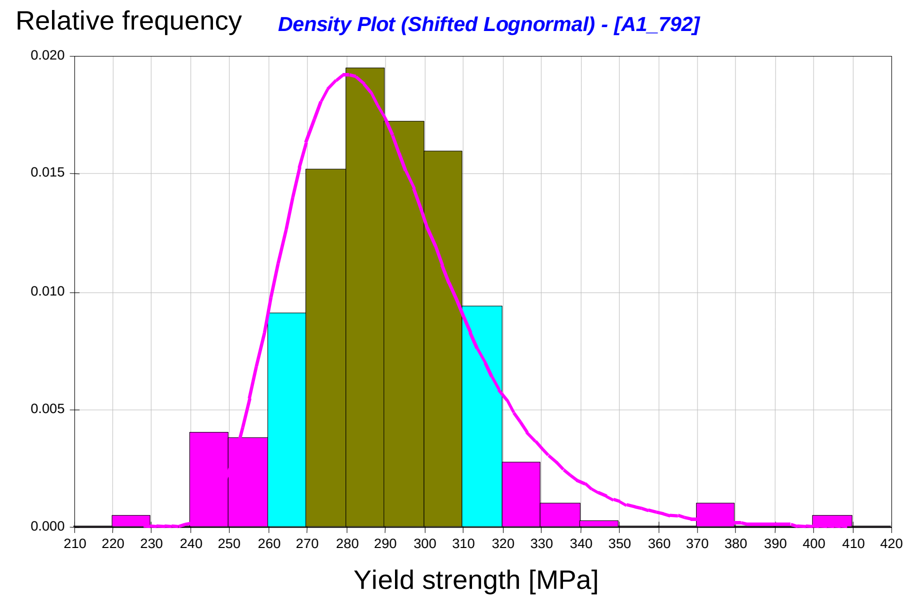
- most important measure of structural reliability

Limit State Approach

- Limit states - states beyond which the structure no longer fulfils the relevant design criteria

- Ultimate limit states
  - loss of equilibrium of a structure as a rigid body
  - rupture, collapse, failure
  - fatigue failure
- Serviceability limit states
  - functional ability of a structure or its part
  - users comfort
  - appearance

Uncertainties



- randomness - natural variability
- statistical uncertainties - lack of data
- model uncertainties - simplified models
- vagueness - imprecision in definitions
- gross errors - human factors
- ignorance - lack of knowledge

EXAMPLE

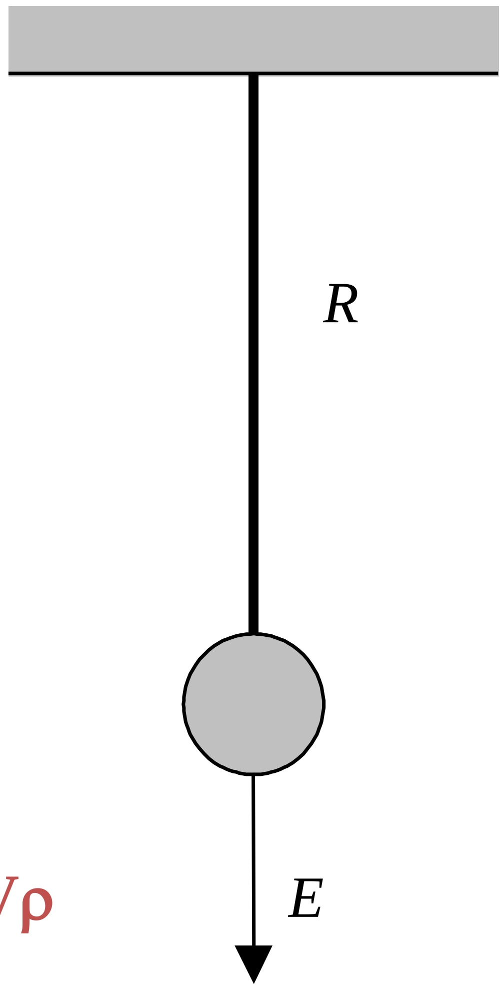
Resistance: $R = \pi d^2 f_y / 4$

Load effect: $E = V\rho$

Failure if E>R or: $V\rho > \pi d^2 f_y / 4$

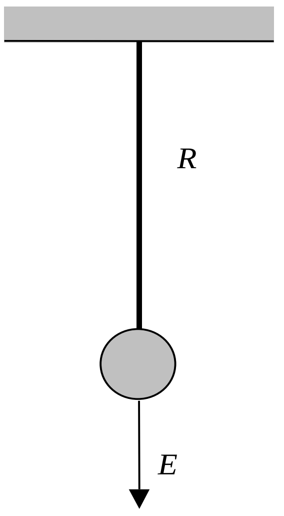
Limit state: $V\rho = \pi d^2 f_y / 4$

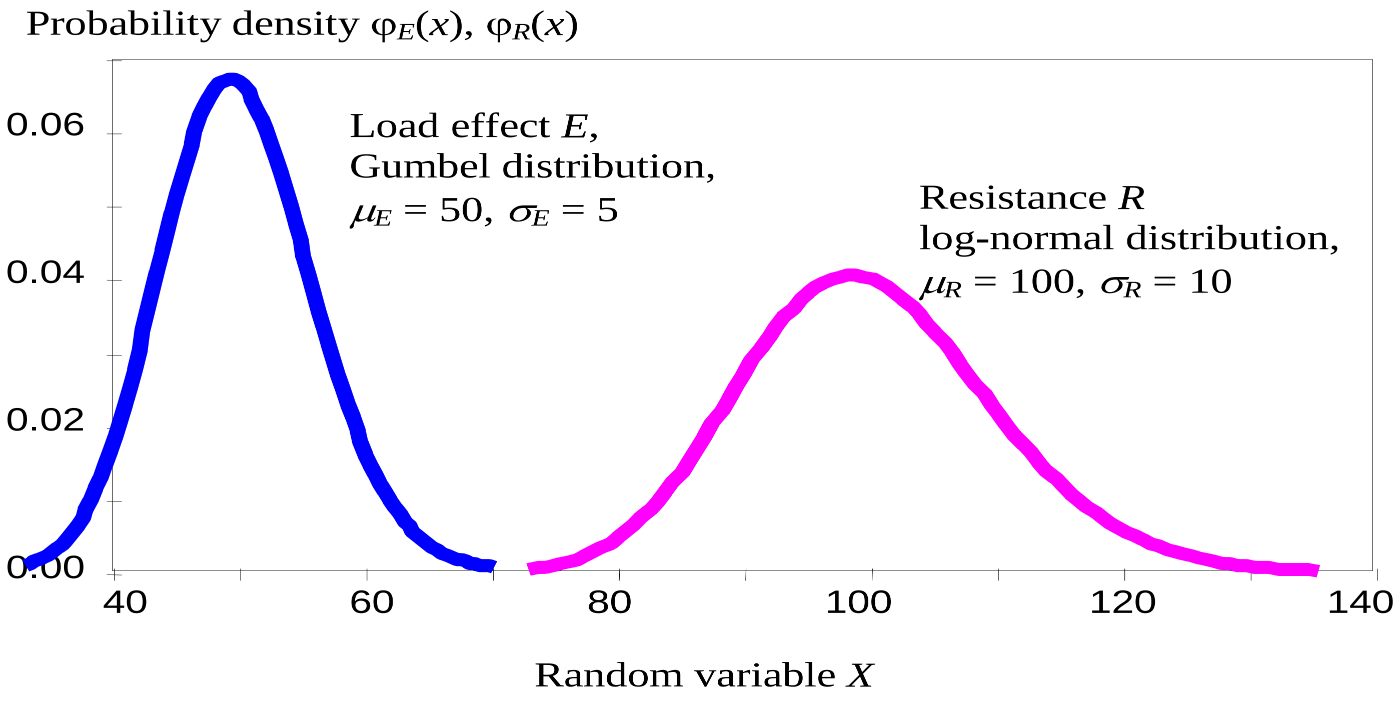
Limit state function: $Z = R - E = \pi d^2 f_y / 4 - V\rho$



Statistical models

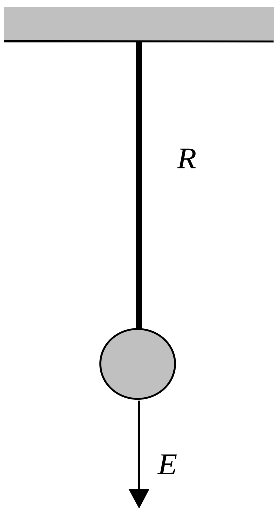
		distribution	mean	sd
R	resistance	Lognormal	100	10
E	load effect	Gumbel	50	5

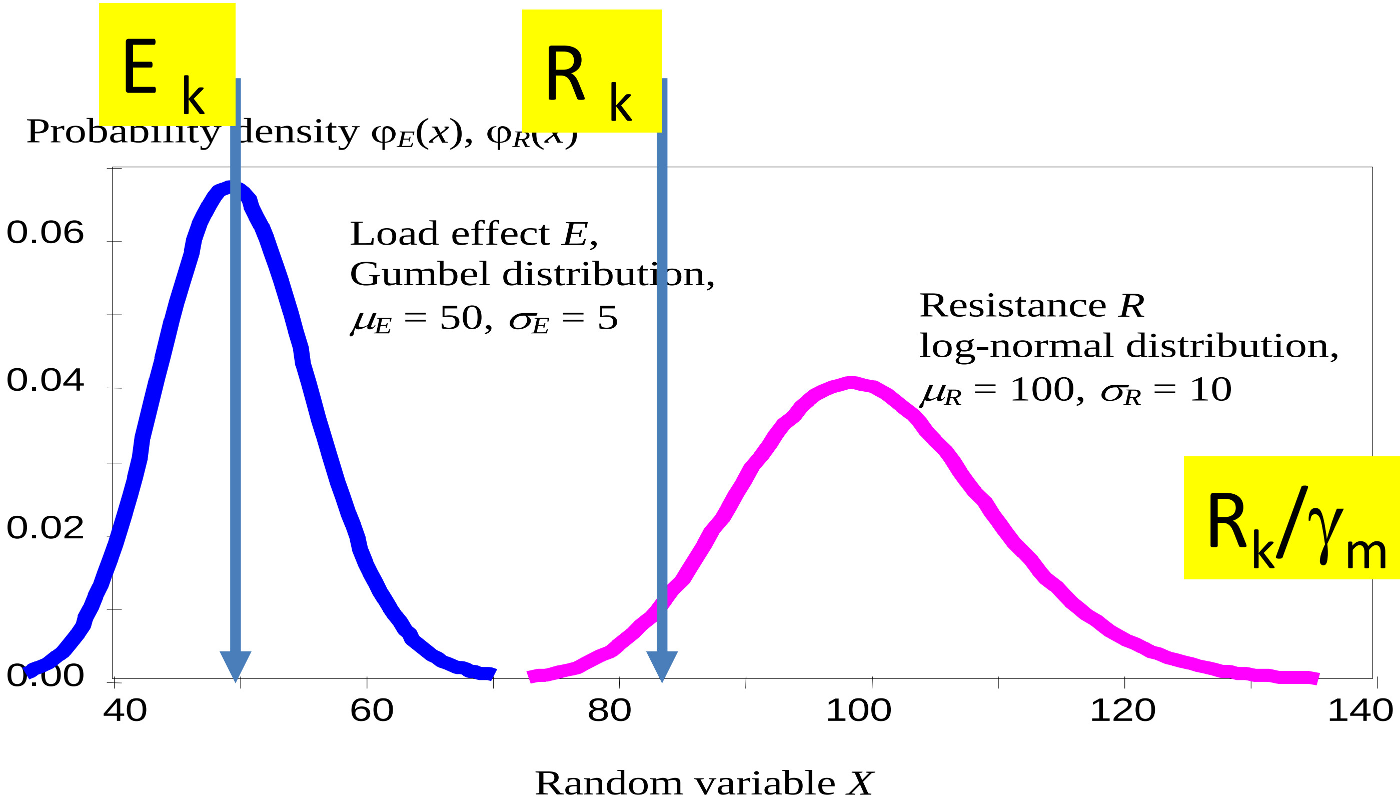




Partial factor approach

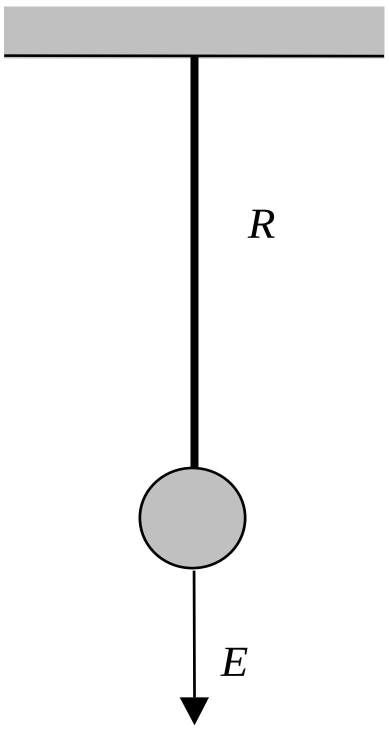
		distribution	mean	sd
R	resistance	Lognormal	100	10
E	load effect	Gumbel	50	5





$R_k/\gamma_m > E_k\,\gamma_Q$

Probabilistic approach



Z = R - E

$$P_f = P(Z<0) = \iint_{Z(X)<0} \varphi_R(r)\varphi_E(e)\,dr\,de$$

Techniques:

Numerical integration (NI)

Monte Carlo (MC)

First order Second moment method (FOSM)

Third moment method (accounting for skewness)

First Order Reliability Methods (FORM)

First Order Second Moment method

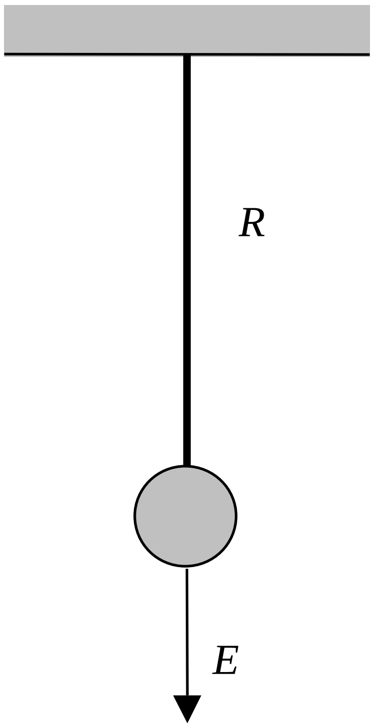
$Z = R - E$

$\mu_Z = \mu_R - \mu_E = \quad 100 - 50 = 50$

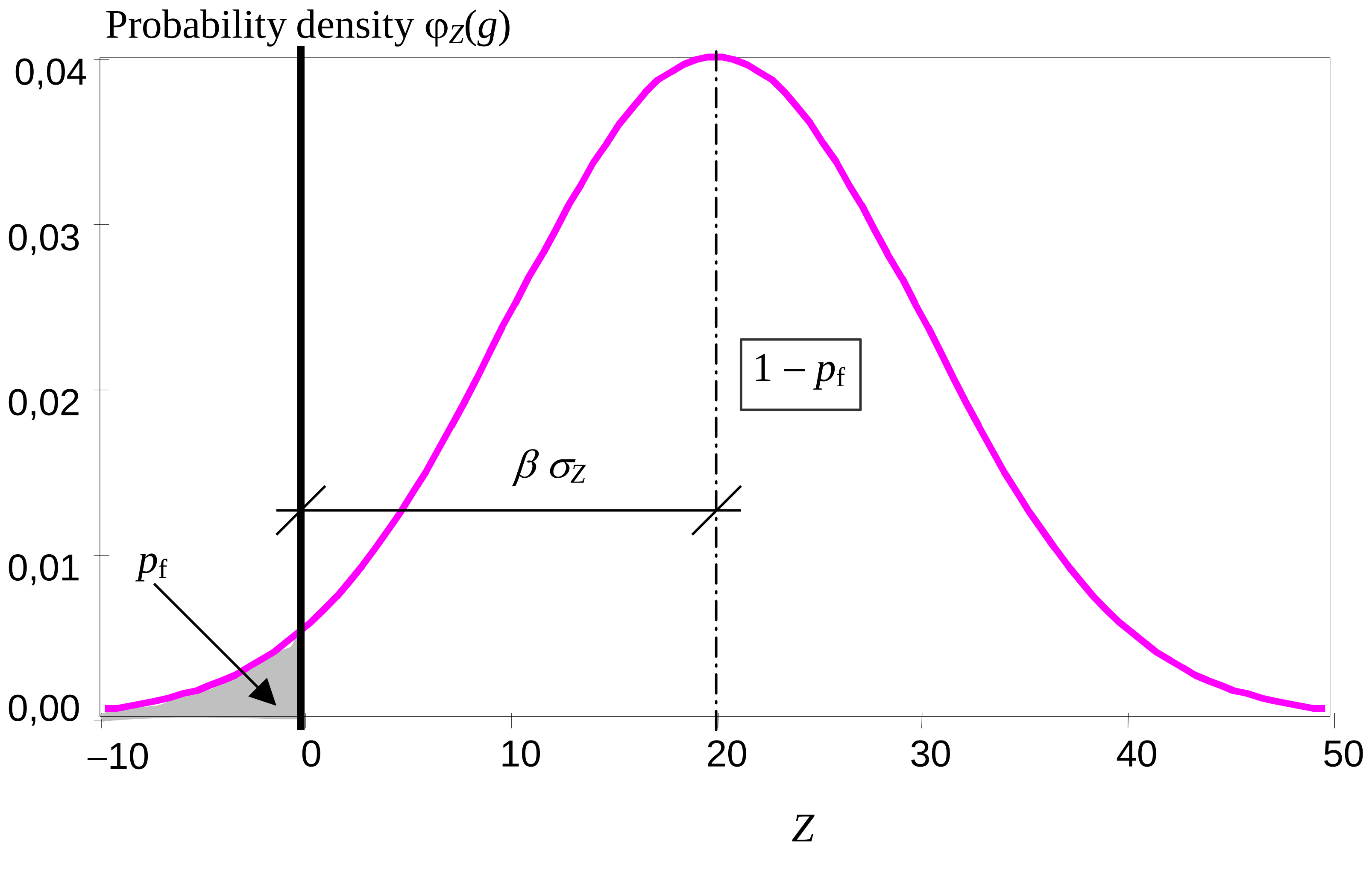
$\sigma_Z^2 = \sigma_R^2 + \sigma_E^2 = 14^2$

$\beta = \mu_Z / \sigma_Z = 3.54$

$P_f = P(Z < 0) = \Phi_Z(0) = 0.0002$



$\beta = \mu_Z / \sigma_Z = 3.54$ $P_f = P(Z < 0) = \Phi_Z(0) = 0.0002$



Reliability index

Probability of Failure = $\Phi(-\beta) \approx 10^{-\beta}$

β	1.3	2.3	3.1	3.7	4.2	4.7
$P(F)=\Phi(-\beta)$	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}

Relation Partial factors and beta-level:

$$\gamma = \exp\{\alpha\, \beta\, V - kV\} \approx 1 + \alpha\, \beta\, V$$

α = 0.7-0.8

β = 3.3 - 3.8 - 4,3 (life time, Annex B)

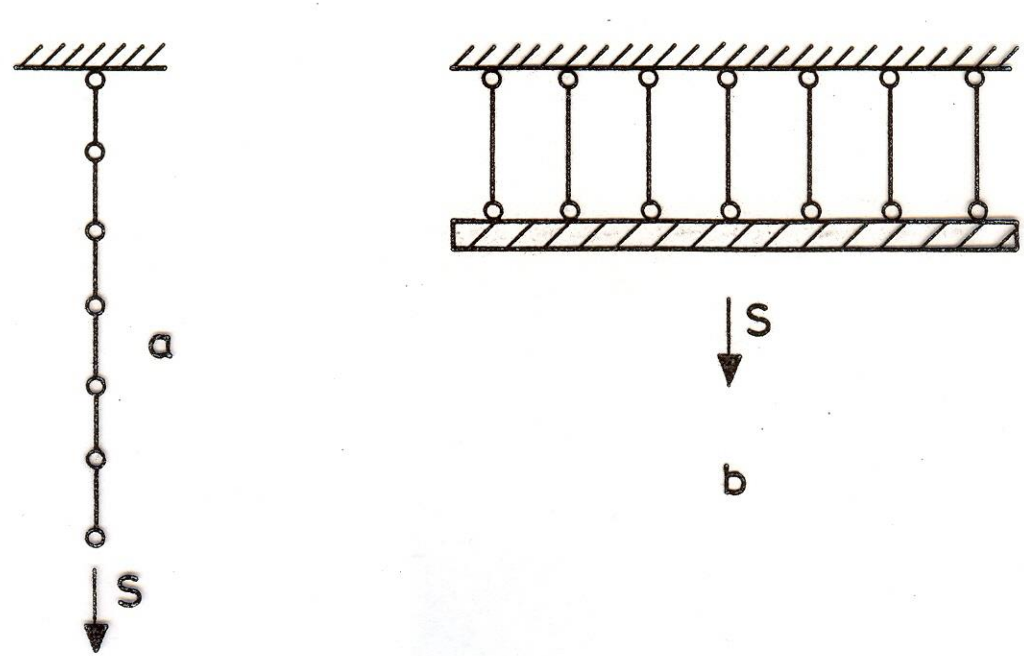
k = 1.64 (resistance)

k = 0.0 (loads)

V = coefficient of variation

Extensions

- load fluctuations
- systems
- degradation
- inspection
- risk analysis
- target reliabilities



Target levels Reliablity

Eurocode EN 1990, Annex B

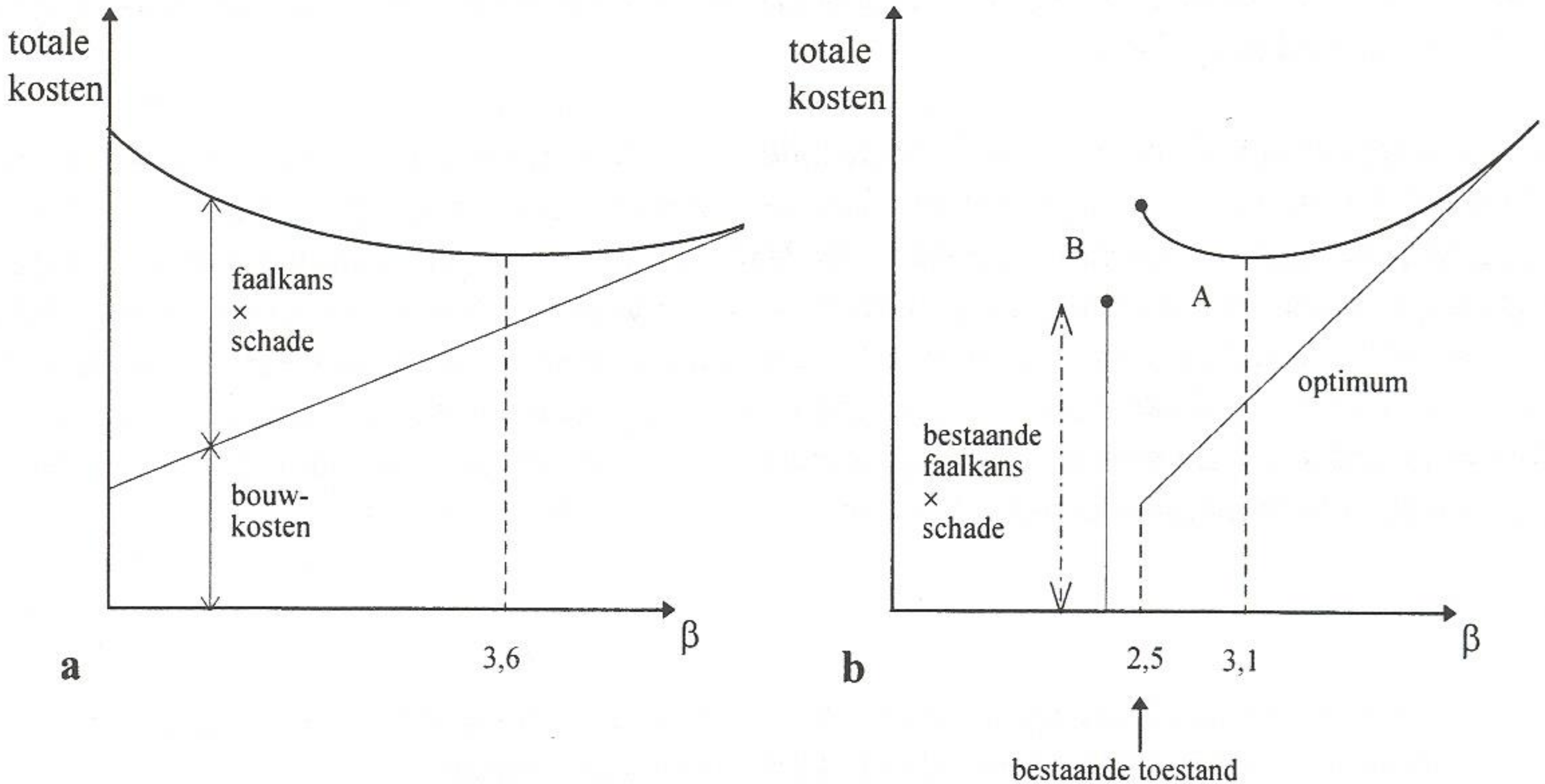
Reliability classes	Consequences for loss of human life, economical, social and environmental consequences	Reliability index β		Examples of buildings and civil engineering works
		β_a for T_a= 1 yr	β_d for T_d= 50 yr	
RC3 – high	High	5,2	4,3	Important bridges, public buildings
RC2 – normal	Medium	4,7	3,8	Residential and office buildings
RC1 – low	Low	4,2	3,3	Agricultural buildings, greenhouses

$$\gamma = \exp\,[(\alpha\beta\text{-}k)V] \sim 1 + \alpha\beta V$$

JCSS TARGET RELIABILITIES β
for a one year reference period

	Consequences of failure ⇒		
Cost to increase safety ⇓	Minor	Moderate	Large
Large	β=3.1 ($p_F \approx 10^{-3}$)	β=3.3 ($p_F \approx 5\ 10^{-4}$)	β=3.7 ($p_F \approx 10^{-4}$)
Normal	β=3.7 ($p_F \approx 10^{-4}$)	β=4.2 ($p_F \approx 10^{-5}$)	β=4.4 ($p_F \approx 5\ 10^{-6}$)
Small	β=4.2 ($p_F \approx 10^{-5}$)	β=4.4 ($p_F \approx 5\ 10^{-5}$)	β=4.7 ($p_F \approx 10^{-6}$)

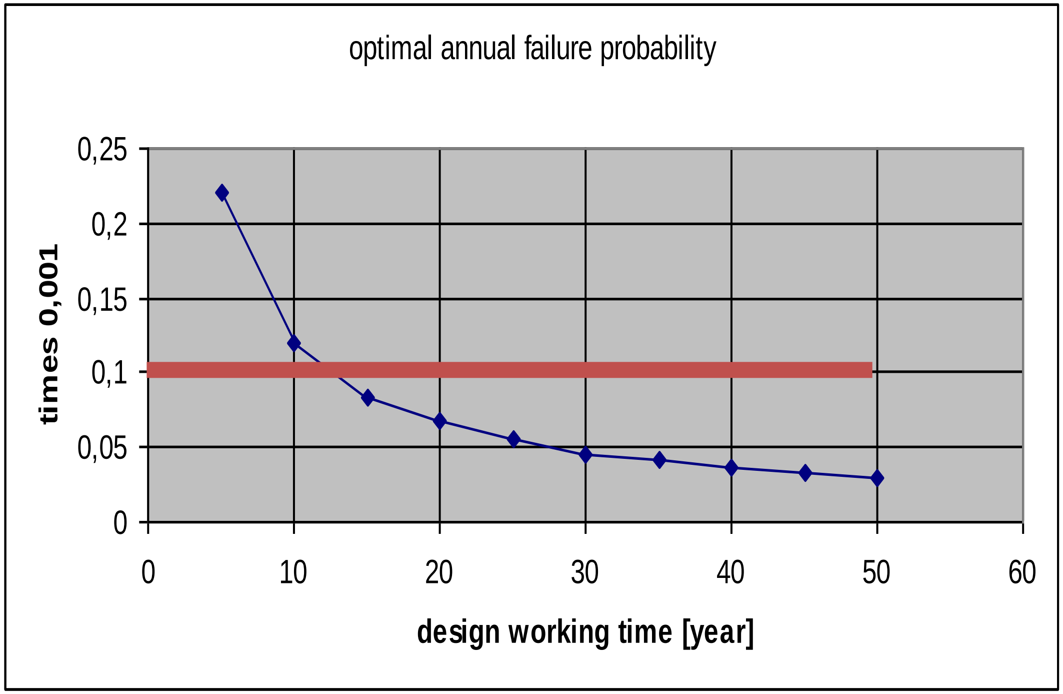
Cost optimisation / design versus assessment



$$P_F = 10^{-\beta}$$

Human life safety

- Include value for human life in *D*
- Still reasons for IR and SR
- Example: $p < 10^{-4}$ / year



Existing Structures (NEN 8700)

Reliability index in case of assessment

Minimum $\beta < \beta_{new} - 1.0$

Human safety: $\beta > 3.6 - 0.8 \log T$

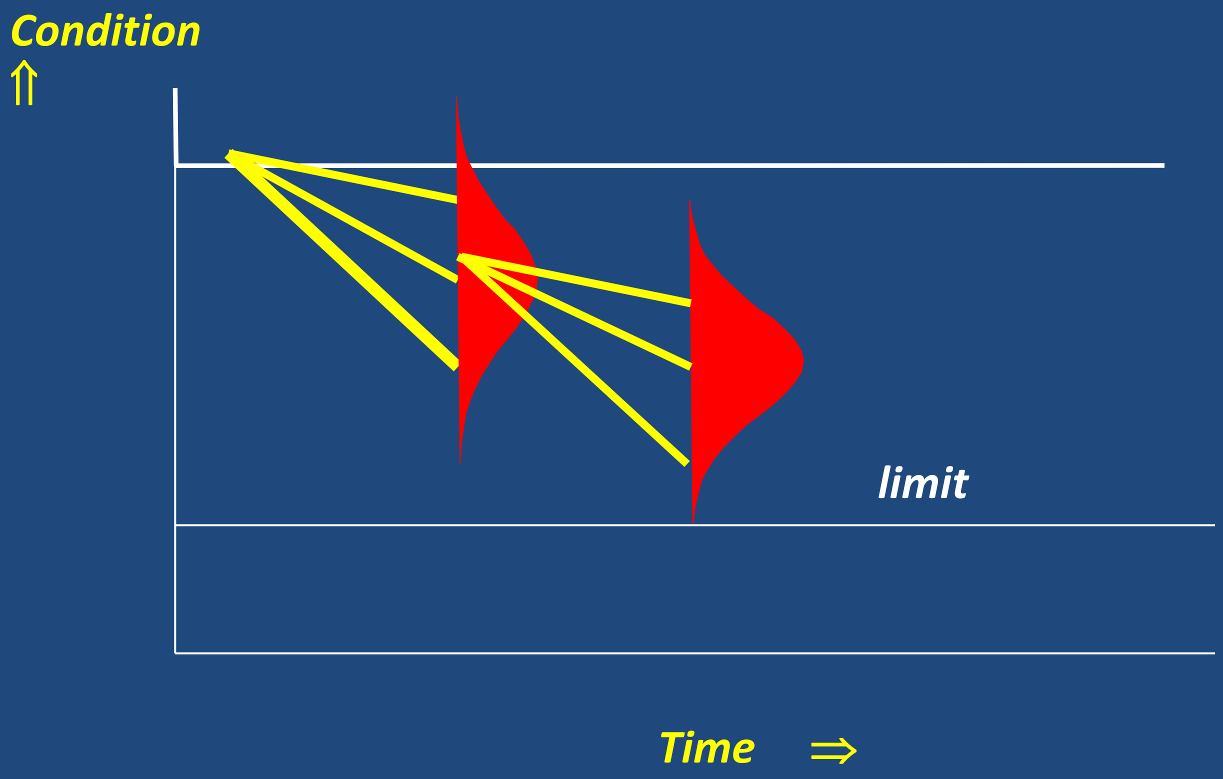
Example NEN 8700 (Netherlands)

Minimum values for the reliability index β with a minimum reference period

Consequence class	Minimum reference period for existing building	β-NEW		β-EXISTING	
		wn	wd	wn	wd
0	1 year	3.3	2,3	1.8	0.8
1	15 years	**3.3**	2,3	1.8[a]	1.1[a]
2	15 years	**3.8**	2.8	2.5[a]	2.5[a]
3	15 years	**4.3**	3.3	3.3[a]	3.3[a]

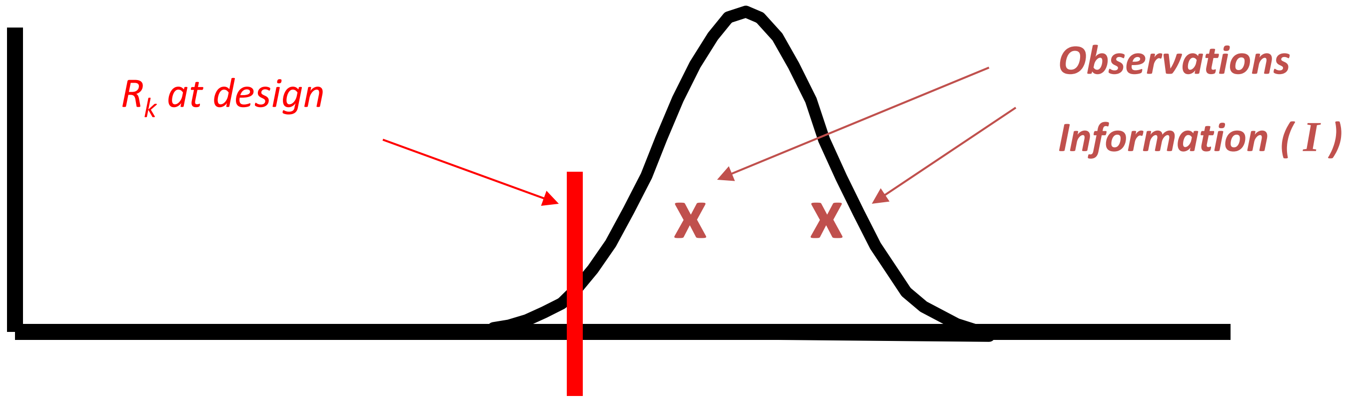
Class 0: As class 1, but no human safety involved
wn = wind not dominant
wd = wind dominant
(a) = in this case is the minimum limit for personal safety normative

Inspection en monitoring



Updating

1) Updating distributions (eg concrete strength)



2) Updating failure probability P{F | I }

Example: I = {crack = 0.6 mm}

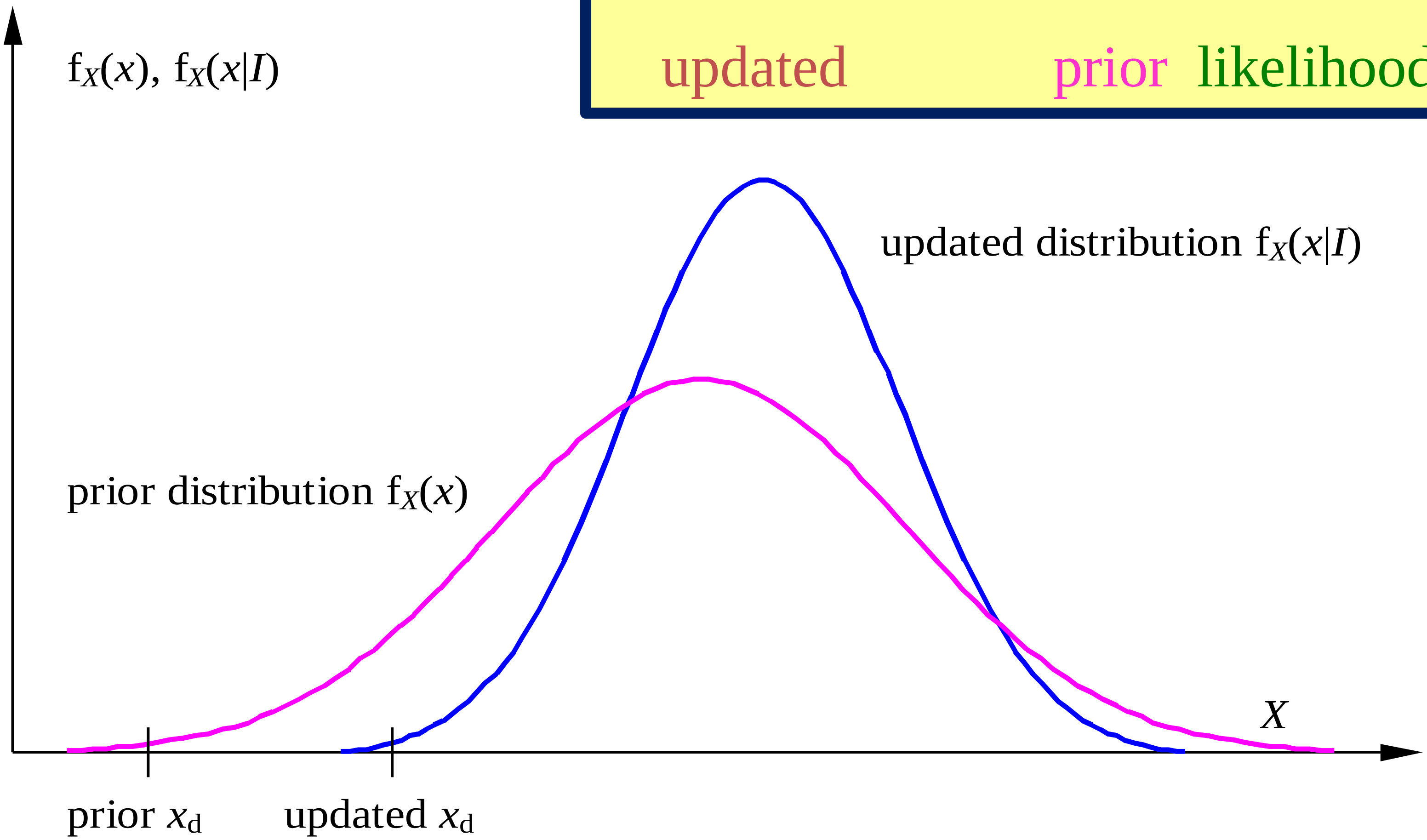
see JCSS document on Existing Structures en ISO13822

Updating distributions

$$P(x|I) = P(x)\,P(I\,|\,x)\,/\,P(I)$$

$$f_X(x|I) = C\;f_X(x)\,P(\,I\,|\,x)$$

updated prior likelihood



Formal Updating formulas

$$f_Q^{''}(q \mid \hat{x}) = C\, f_Q^{'}(q)\, L(\mid \hat{x} \mid q)$$

$$f_X^U(x) = \int_{-\infty}^{\infty} f_X(x|q)\, f_Q^{''}(q \mid \hat{x})\, dq$$

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Ask the expert !

Example: Resistance with unknown mean m_R and known stand. Dev. s_R =17,5

Assume we have 3 observations with mean m_m = 350
Then m_R has $s_m = 17{,}5/\sqrt{3} = 10$.
If the load is to 304 then:

m_Z= 350-304=46
$s_Z = \sqrt{(17{,}5^2 + 10^2)} = 20{,}2$
β=2,27
P_f=0,0116

Now we have one extra observation equal to 350.
In that case the estimate of the mean m_m does not change.
The standard deviation of the mean changes to $17{,}5/\sqrt{(4)}$ = 8,8

m_Z= 350-304=46,
$s_Z = \sqrt{(17{,}5^2 + 8{,}8^2)} = 19{,}6$,
β=2,35
P_f=0,0095

Summary Reliability aspects

- Uncertainties exist
- Probability Theory may be helpful
- Reliability targets depends on consequences of failure
- Reliability targets depend on costs of improving
- Existing structures may have a lower target reliability
- Reliability may be updated using inspection results
- There is a relation partial factor – reliability index