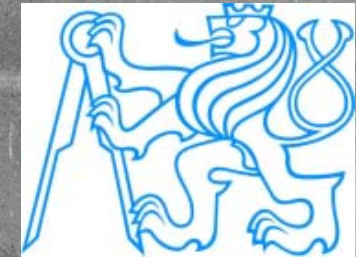


Lifelong  
Learning  
Programme



# Reliability Aspects

Milan Holický  
CTU in Prague

Leonardo da Vinci

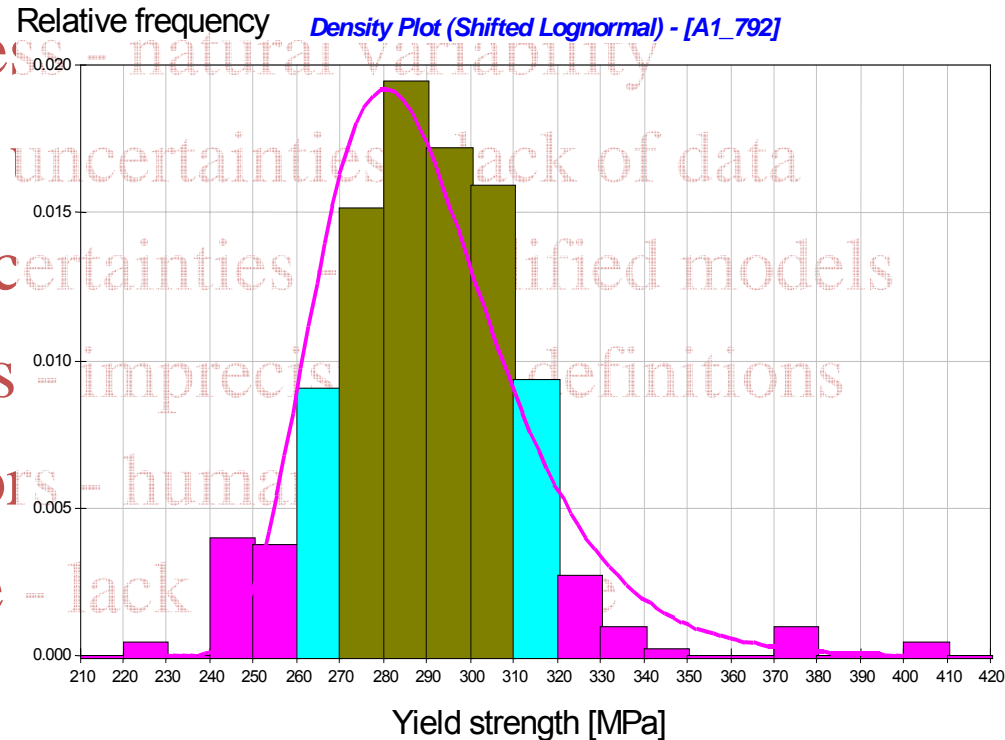
Assessment of existing structures

Project number: CZ/08/LLP-LdV/TOI/134005

# Uncertainties are always present

## •Uncertainties (aleatoric and epistemic) Description

- randomness - natural variability
- statistical uncertainties - lack of data
- model uncertainties - simplified models
- vagueness - imprecise definitions
- gross errors - human
- ignorance - lack



## •Tools

- theory of probability and statistics, fuzzy logic
- reliability theory and risk engineering

Some uncertainties are difficult to quantify

# Eurocode EN 1990:

## Reliability

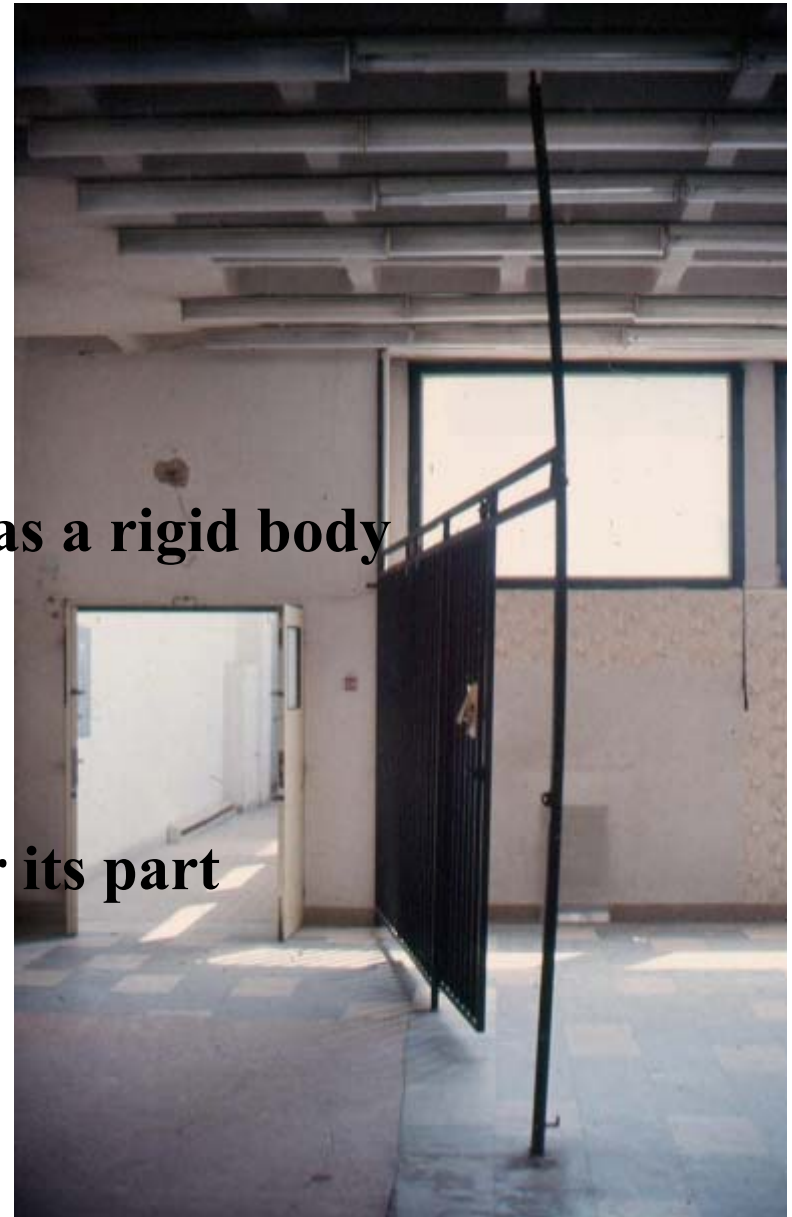
- ability of a structure to fulfil all required functions during a specified period of time under given conditions

## Failure probability $P_f$

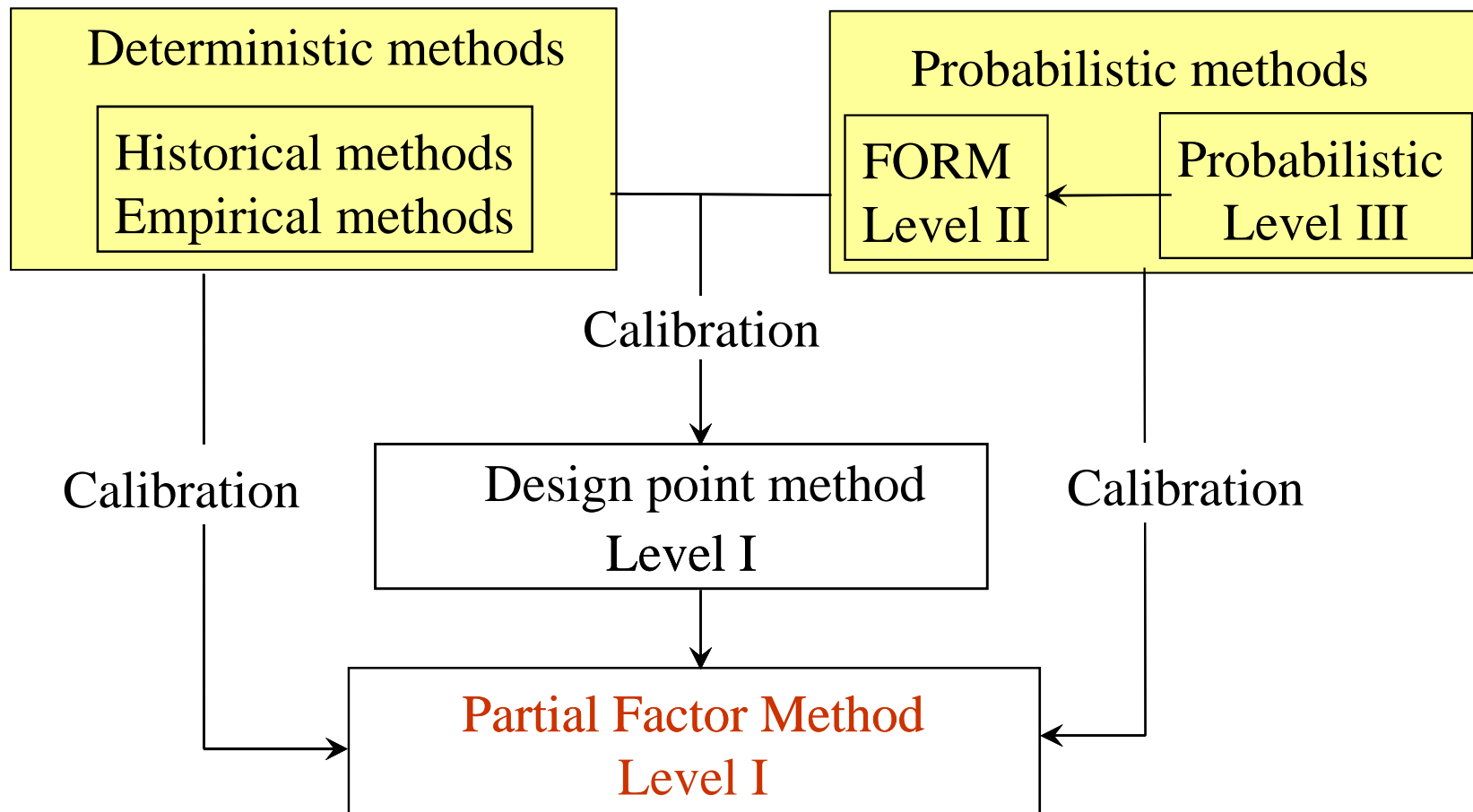
- most important measure of structural reliability

# Limit State Approach

- **Limit states** - states beyond which the structure no longer fulfils the relevant design criteria
- **Ultimate limit states**
  - loss of equilibrium of a structure as a rigid body
  - rupture, collapse, failure
  - fatigue failure
- **Serviceability limit states**
  - functional ability of a structure or its part
  - users comfort
  - appearance



# Reliability Methods



Reliability measures: failure probability  $p_f$  and reliability index  $\beta$

$p_f$	$10^{-1}$	$10^{-2}$	$10^{-3}$	<b><math>10^{-4}</math></b>	$10^{-5}$	$10^{-6}$	$10^{-7}$
$\beta$	1,3	2,3	3,1	<b>3,7</b>	4,2	4,7	5,2

# Fundamental case for normal distribution

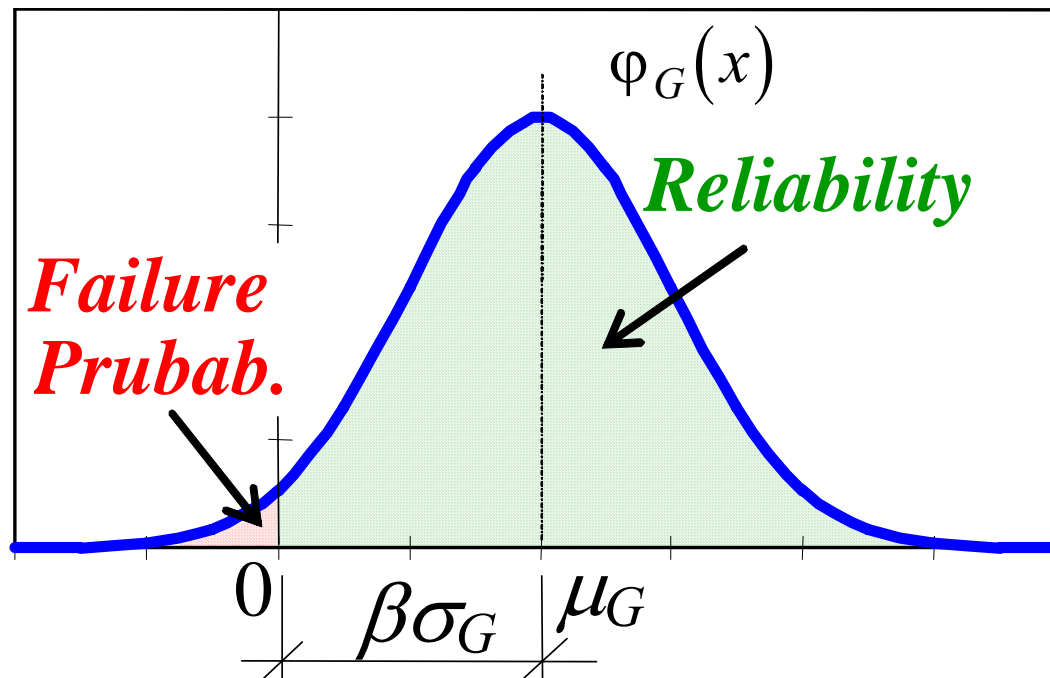
$$E \leq R$$

$$G = R - E \quad \mu_G = \mu_R - \mu_E, \quad \sigma_G^2 = \sigma_R^2 + \sigma_E^2$$

Transformation of  $G$  to standardized variable  $U = (G - \mu_G) / \sigma_G$

For  $G = 0$  the standardized variable  $u_0 = (0 - \mu_G) / \sigma_G$

Reliability index : 
$$\beta = -u_0 = \frac{\mu_G}{\sigma_G} = \frac{\mu_R - \mu_E}{(\sigma_R^2 + \sigma_E^2)^{1/2}}$$



Failure probability

$$p_f = \Phi(-\beta)$$

# Fundamental case $E < R$

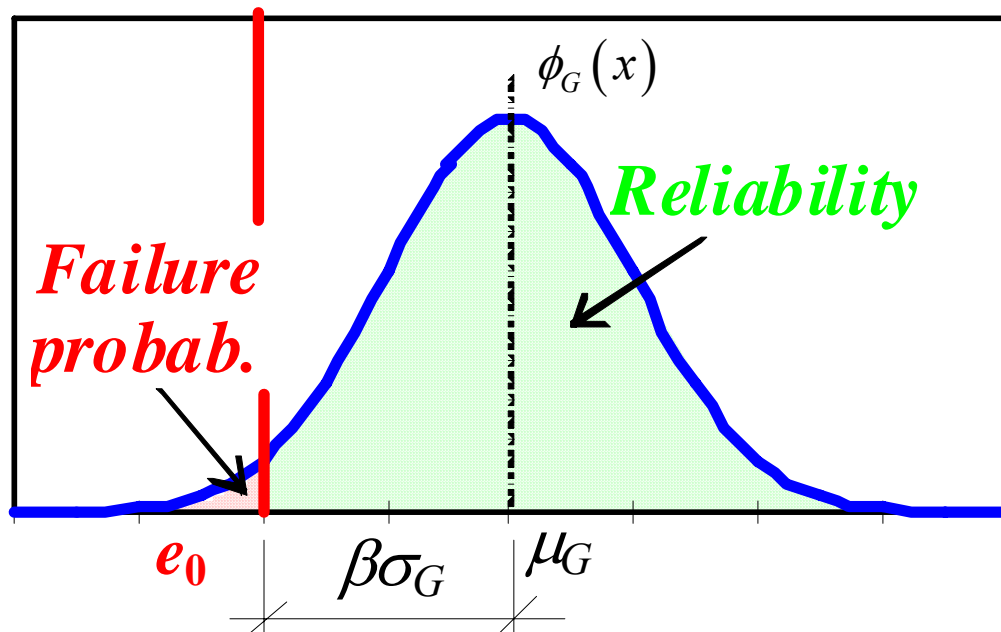
Limit state function:  $g(X) = G = R - E = 0$

Load  $E=e_0$  known, resistance  $R$  random:  $\mu_R, \sigma_R, (\alpha_R)$

Transformation of  $R$  to standardized variable  $U=(R - \mu_R)/\sigma_R$

For  $R = e_0$  the standardized variable  $u_0 = (e_0 - \mu_R)/\sigma_R$

Reliability index for normal dist.:  $\beta = -u_0 = (\mu_R - e_0) / \sigma_R$



Failure probability

$$p_f = \Phi_R(e_0)$$

For normal distribution

$$p_f = \Phi(-\beta)$$

# An example of the fundamental case

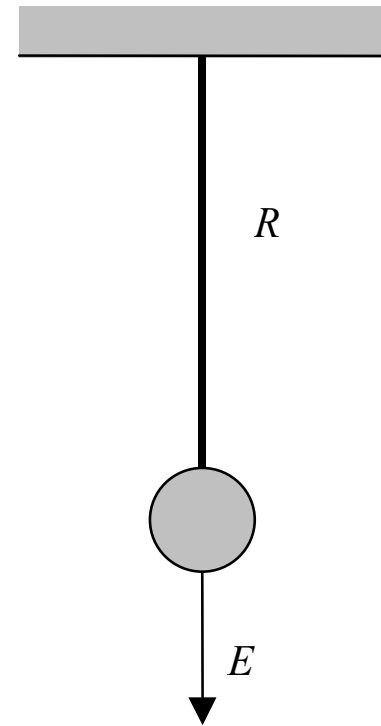
$$Z = R - E$$

$$\mu_Z = \mu_R - \mu_E = 100 - 50 = 50$$

$$\sigma_Z^2 = \sigma_R^2 + \sigma_E^2 = 14^2$$

$$\beta = \mu_Z / \sigma_Z = 3.54$$

$$P_f = P(Z < 0) = \Phi_Z(0) = 0.0002$$



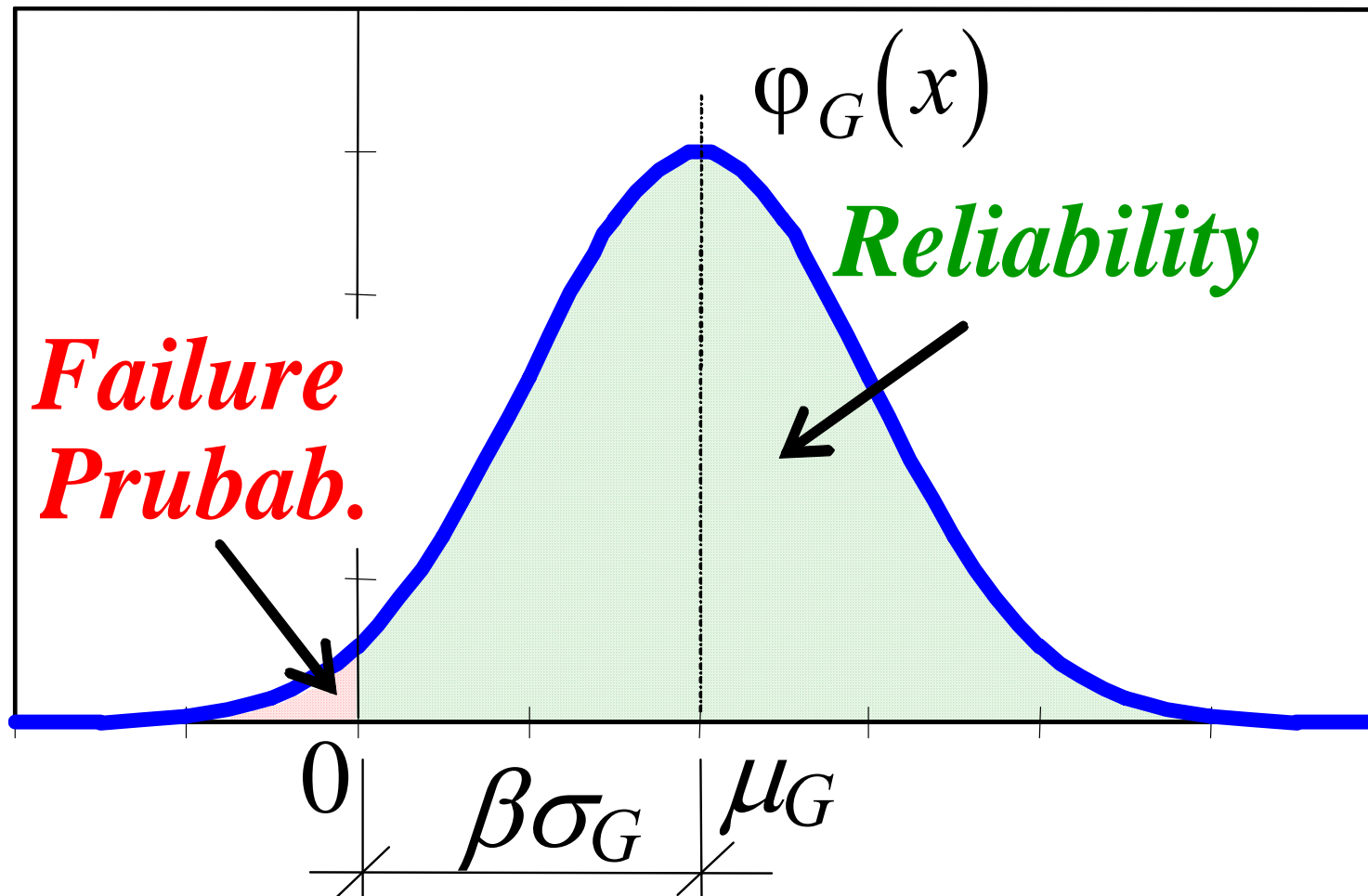


# Reliability margin and index $\beta$

$$\beta = \mu_Z / \sigma_Z = 3.54$$

$$P_f = P(Z < 0) = \Phi_Z(0) = 0.0002,$$

$\beta$  Is the distance of the mean of reliability margin from the origin



# Probabilistic approach

$$Z = R - E$$

$$P_f = P(Z < 0) = \iint_{Z(\mathbf{X}) < 0} \varphi_R(r) \varphi_E(e) dr de$$

Techniques:

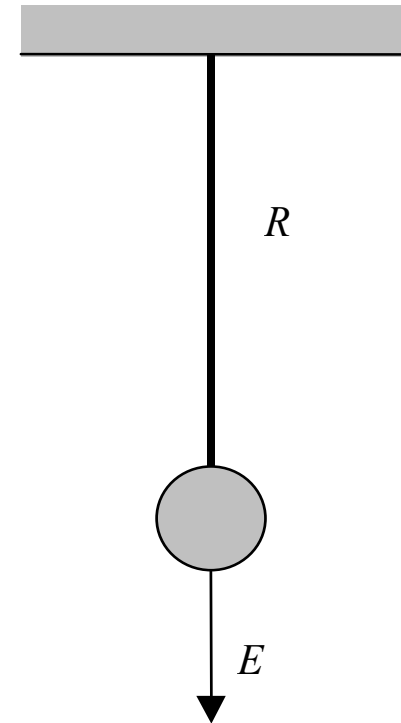
Numerical integration (NI)

Monte Carlo (MC)

First order Second moment method (FOSM)

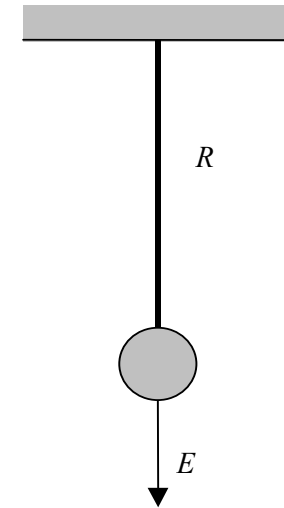
Third moment method (accounting for skewness)

First Order Reliability Methods (FORM)

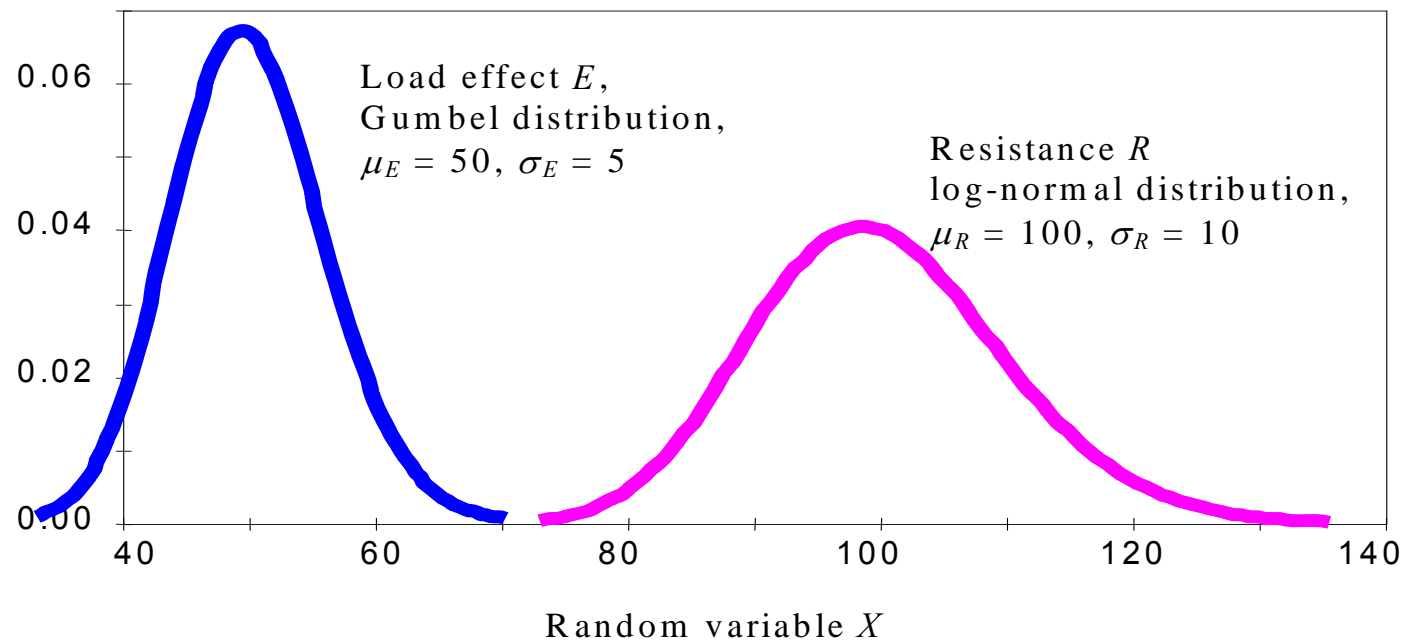


# Probabilistic models

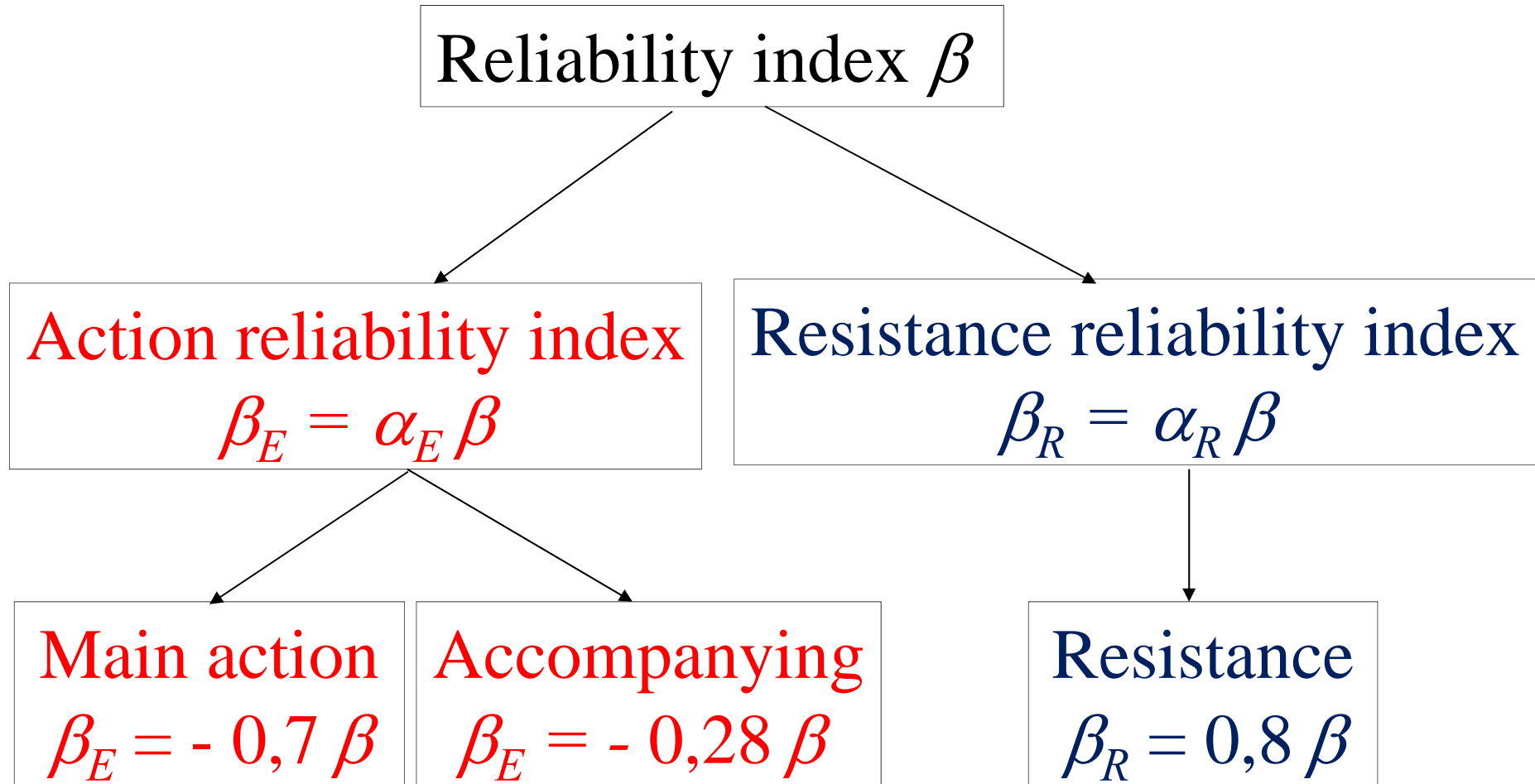
		<b>distribution</b>	<b>mean</b>	<b>sd</b>
R	resistance	Lognormal	100	10
E	load effect	Gumbel	50	5



Probability density  $\varphi_E(x)$ ,  $\varphi_R(x)$



# Eurocode concepts of partial factors



# Partial factor

- Design value

for normal and lognormal distribution

$$x_d = \mu(1 - \alpha\beta V)$$

for lognormal distribution:  $x_d$

$$= \mu \exp(-\alpha\beta \sigma - 0.5 \sigma^2)$$

- Characteristic value

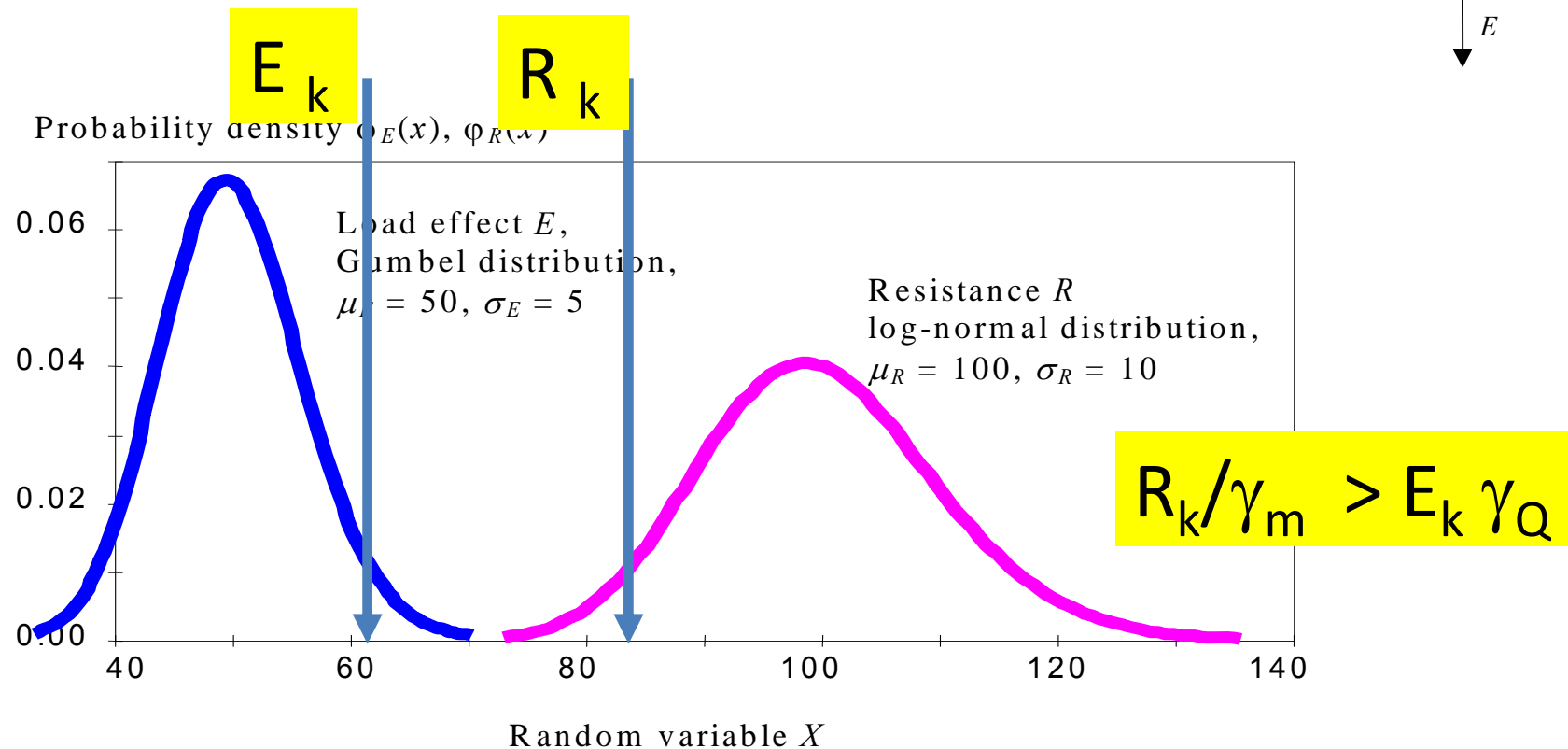
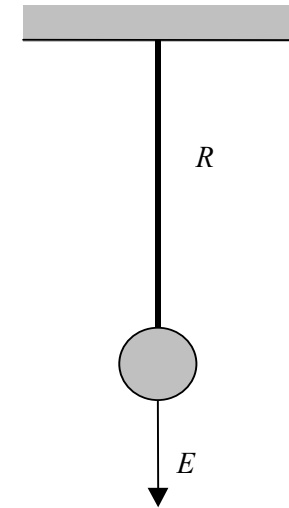
for normal  $x_k = \mu(1 - kV)$

for lognormal  $x_k = \mu \exp(-k \sigma - 0.5 \sigma^2)$

- Partial factor  $\gamma_m = \frac{x_k}{x_d}$

# Partial factor approach

		<b>distribution</b>	<b>mean</b>	<b>sd</b>
R	resistance	Lognormal	100	10
E	load effect	Gumbel	50	5



# Indicative target reliabilities in ISO 13822

Limit states	Target reliability index, $\beta$	Reference period
Serviceability		
Reversible	0,0	Intended remaining working life
Irreversible	1,5	Intended remaining working life
Fatigue		
inspectable	2,3	Intended remaining working life
not inspectable	3,1	Intended remaining working life
Ultimate		
very low consequences of failure	2,3	$L_S$ years*
low consequence of failure	3,1	$L_S$ years*
medium consequence of failure	3,8	$L_S$ years*
high consequence of failure	4,3	$L_S$ years*
* $L_S$ is a minimum standard period for safety (e.g. 50 years)		

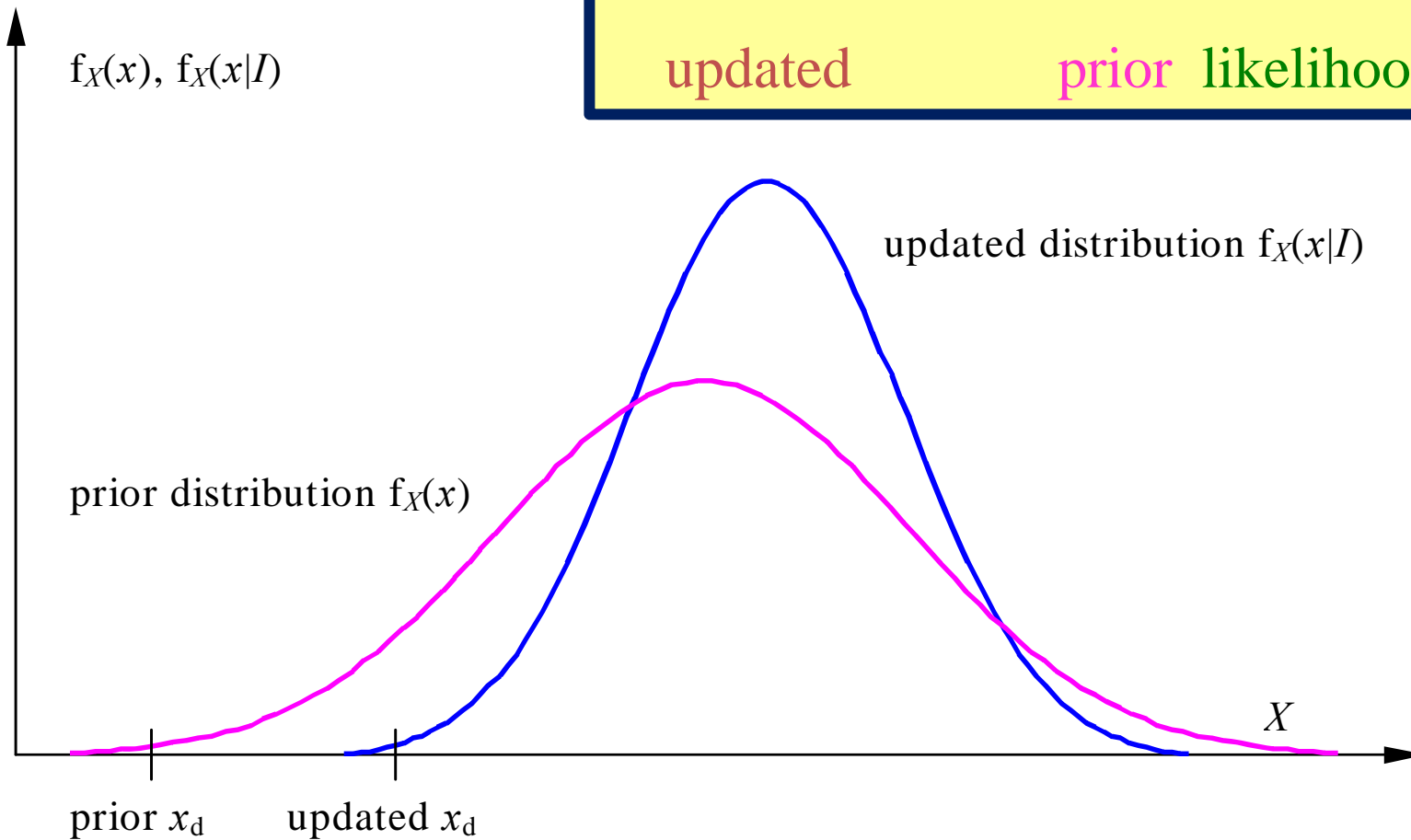
# Updating distributions

$$P(x|I) = P(x) P(I | x) / P(I)$$

$$f_X(x|I) = C f_X(x) P(I | x)$$

updated

prior likelihood





## Formal Updating formulas

$$f_Q''(q | \hat{x}) = C f_Q'(q) L(|\hat{x}| q)$$

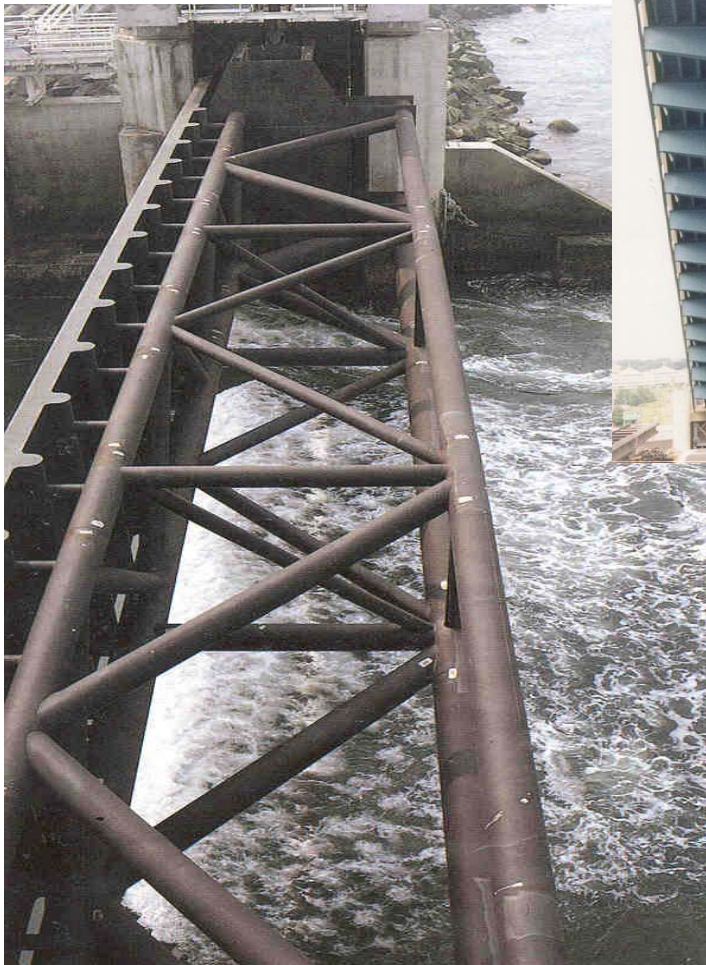
$$f_X^U(x) = \int_{-\infty}^{\infty} f_X(x|q) f_Q''(q | \hat{x}) dq$$

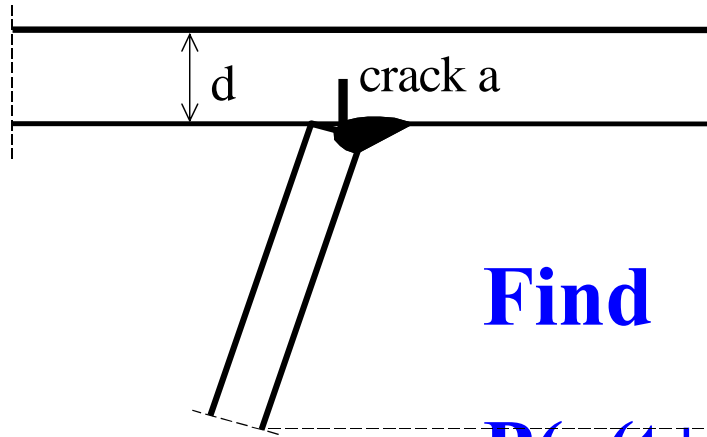
**Ask the expert !**

# Concluding remarks on reliability aspects

- ❑ Uncertainties are always present
- ❑ Probability Theory may be helpful
- ❑ Reliability targets depends on consequences of failure
- ❑ Reliability targets depend on costs of improving
- ❑ Existing structures may have a lower target reliability
- ❑ Reliability may be updated using inspection results
- ❑ There is a relation partial factor – reliability index

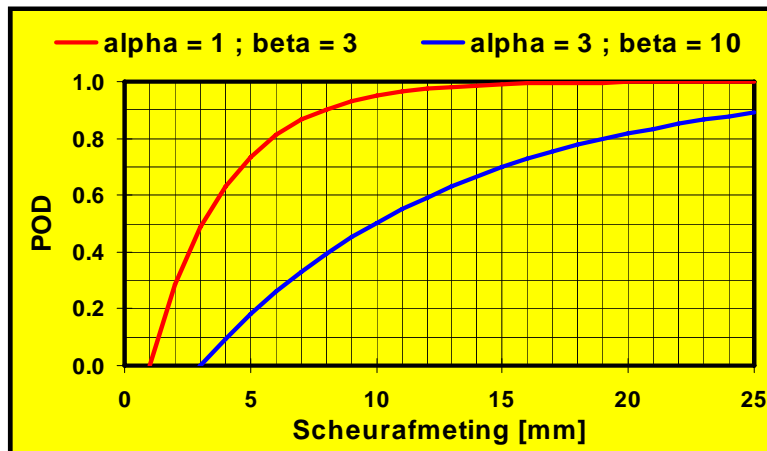
# Fatigue steel structures



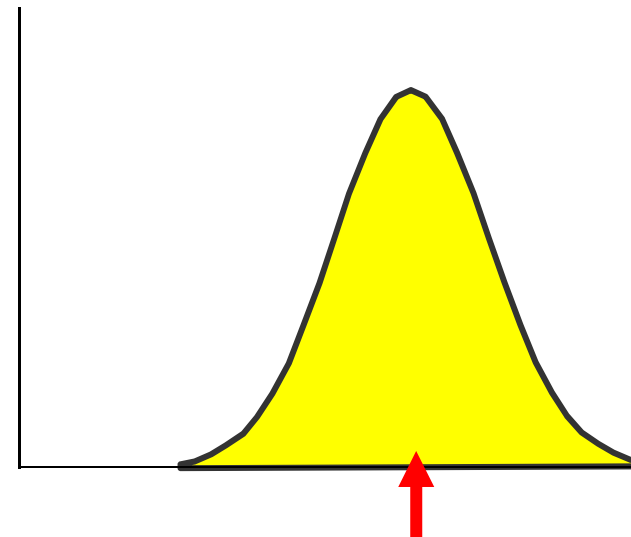


**Find**

$$P(a(t+\Delta t) > d \mid a(t) = \dots \text{ of } a(t) < \dots)$$



**no cracks found, but?**



**measured 1 mm, but?**

**Example:** Resistance with unknown mean  $m_R$  and known stand. Dev.  $s_R = 17,5$

Assume we have 3 observations with mean  $m_m = 350$

Then  $m_R$  has  $s_m = 17,5/\sqrt{3} = 10$ .

If the load is to 304 then:

$$\begin{aligned}m_z &= 350 - 304 = 46 \\s_z &= \sqrt{(17,5^2 + 10^2)} = 20,2 \\ \beta &= 2,27 \\ P_f &= 0,0116\end{aligned}$$

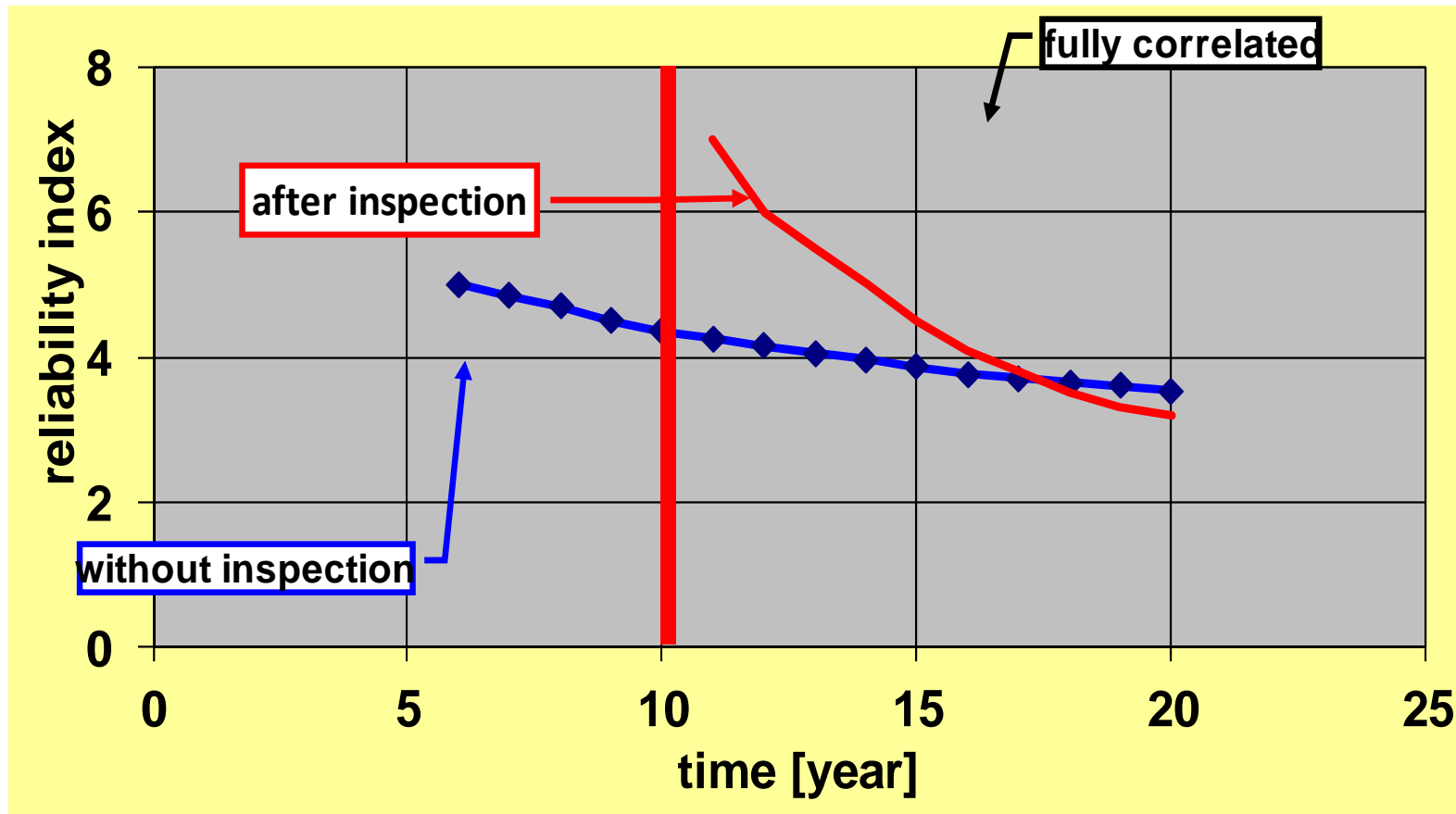
Now we have one extra observation equal to 350.

In that case the estimate of the mean  $m_m$  does not change.

The standard deviation of the mean changes to  $17,5/\sqrt{4} = 8,8$

$$\begin{aligned}m_z &= 350 - 304 = 46, \\s_z &= \sqrt{(17,5^2 + 8,8^2)} = 19,6, \\ \beta &= 2,35 \\ P_f &= 0,0095\end{aligned}$$

Reliability level Beta (one year periods)  
given a crack found at  $t=10$  a



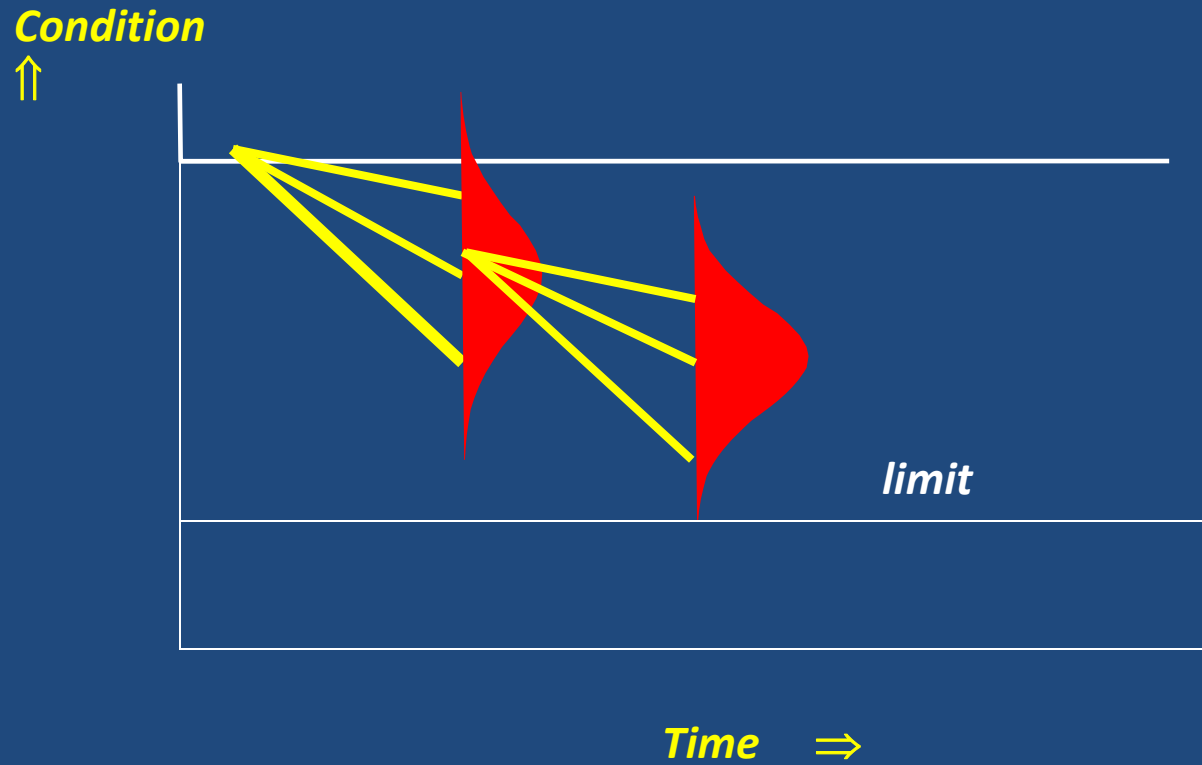
## Existing Structures (NEN 8700)

### Reliability index in case of assessment

Minimum  $\beta < \beta_{\text{new}} - 1.0$

Human safety:  $\beta > 3.6 - 0.8 \log T$

# Inspection en monitoring





# EXAMPLE

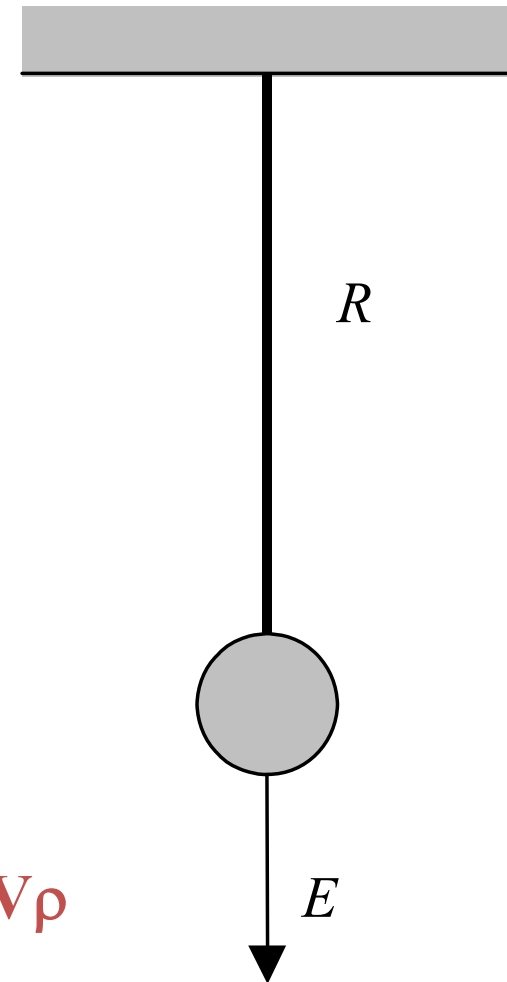
**Resistance:**  $R = \pi d^2 f_y / 4$

**Load effect:**  $E = V\rho$

**Failure if  $E > R$  or:**  $V\rho > \pi d^2 f_y / 4$

**Limit state:**  $V\rho = \pi d^2 f_y / 4$

**Limit state function:**  $Z = R - E = \pi d^2 f_y / 4 - V\rho$



# Reliability index

Probability of Failure =  $\Phi(-\beta) \approx 10^{-\beta}$

$\beta$	1.3	2.3	3.1	3.7	4.2	4.7
$P(F)=\Phi(-\beta)$	$10^{-1}$	$10^{-2}$	$10^{-3}$	$10^{-4}$	$10^{-5}$	$10^{-6}$

Relation Partial factors and beta-level:

$$\gamma = \exp\{\alpha \beta V - kV\} \approx 1 + \alpha \beta V$$

$$\alpha = 0.7-0.8$$

$$\beta = 3.3 - 3.8 - 4,3 \text{ (life time, Annex B)}$$

$$k = 1.64 \text{ (resistance)}$$

$$k = 0.0 \text{ (loads)}$$

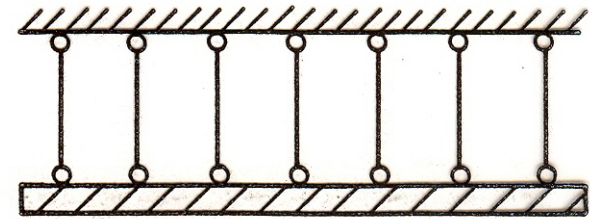
$V$  = coefficient of variation

# Extensions

- load fluctuations
- systems
- degradation
- inspection
- risk analysis
- target reliabilities



a



S

b

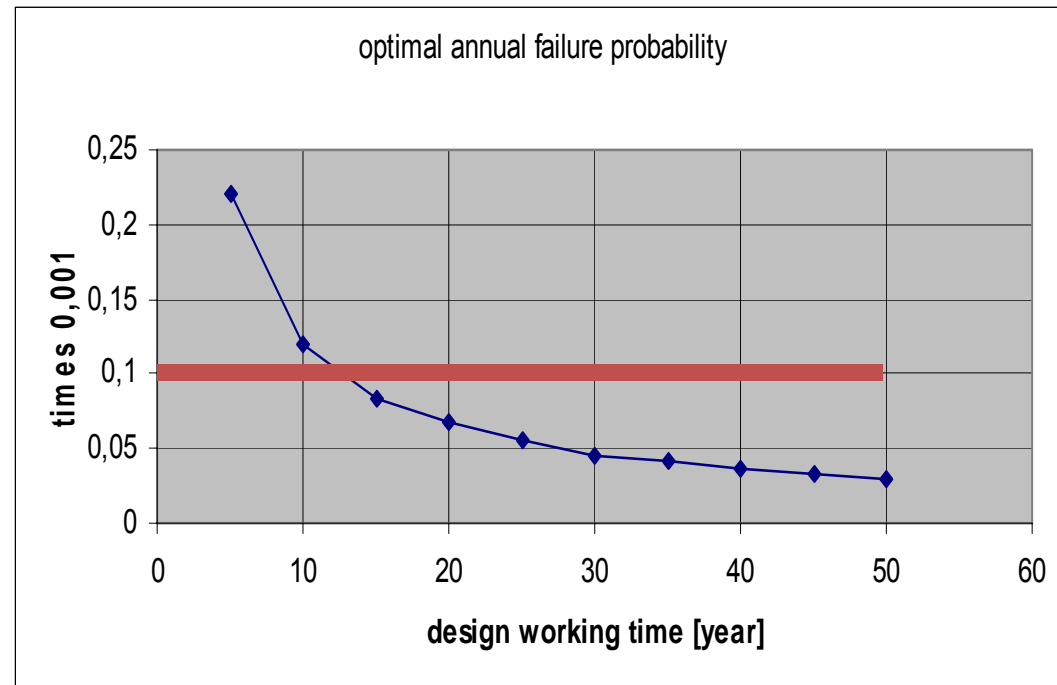


# JCSS TARGET RELIABILITIES $\beta$ for a one year reference period

	Consequences of failure $\Rightarrow$		
Cost to increase safety $\Downarrow$	Minor	Moderate	Large
Large	$\beta=3.1$ ( $p_f \approx 10^{-3}$ )	$\beta=3.3$ ( $p_f \approx 5 \cdot 10^{-4}$ )	$\beta=3.7$ ( $p_f \approx 10^{-4}$ )
Normal	$\beta=3.7$ ( $p_f \approx 10^{-4}$ )	$\beta=4.2$ ( $p_f \approx 10^{-5}$ )	$\beta=4.4$ ( $p_f \approx 5 \cdot 10^{-6}$ )
Small	$\beta=4.2$ ( $p_f \approx 10^{-5}$ )	$\beta=4.4$ ( $p_f \approx 5 \cdot 10^{-5}$ )	$\beta=4.7$ ( $p_f \approx 10^{-6}$ )

# Human life safety

- Include value for human life in  $D$
- Still reasons for IR and SR
- Example:  $p < 10^{-4}$  / year



*Example NEN 8700 (Netherlands)*

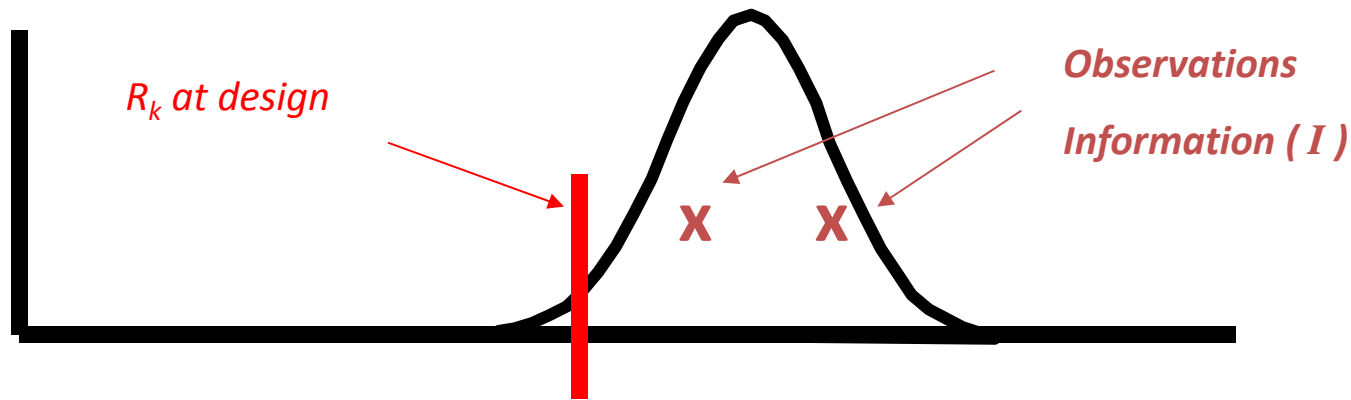
*Minimum values for the reliability index  $\beta$  with a minimum reference period*

Consequence class	Minimum reference period for existing building	$\beta$ -NEW		$\beta$ -EXISTING	
		wn	wd	wn	wd
0	1 year	3.3	2,3	1.8	0.8
1	15 years	<b>3.3</b>	2,3	1.8 <sup>a</sup>	1.1 <sup>a</sup>
2	15 years	<b>3.8</b>	2.8	2.5 <sup>a</sup>	2.5 <sup>a</sup>
3	15 years	<b>4.3</b>	3.3	3.3 <sup>a</sup>	3.3 <sup>a</sup>

Class 0: As class 1, but no human safety involved  
 wn = wind not dominant  
 wd = wind dominant  
 (a) = in this case is the minimum limit for personal safety normative

# Updating

## 1) Updating distributions (eg concrete strength)



## 2) Updating failure probability $P\{F \mid I\}$

Example:  $I = \{\text{crack} = 0.6 \text{ mm}\}$

see JCSS document on Existing Structures en ISO13822



$$P(A \cap B) = P(A|B)P(B)$$

$$P(F \cap I) = P(F|I)P(I)$$

$$P(F|I) = \frac{P(F \cap I)}{P(I)}$$

Two types of information I:

equality type:  $h(\mathbf{x}) = 0$

inequality type:  $h(\mathbf{x}) < 0$ ;  $h(\mathbf{x}) > 0$

$\mathbf{x}$  = vector of basic variables

$$P(F|I) = \frac{P(Z(t_2) < 0 \cap h(t_1) > 0)}{P(h(t_1) > 0)}$$

# Target Reliabilities in EN 1990, Annex B

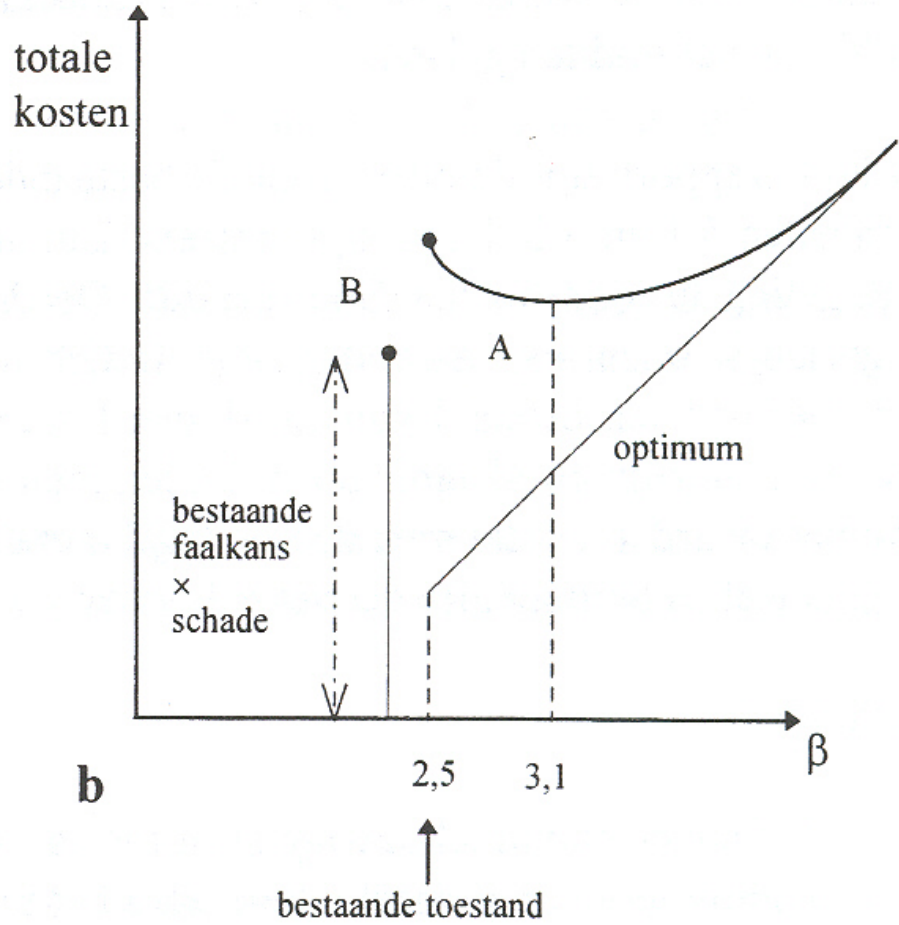
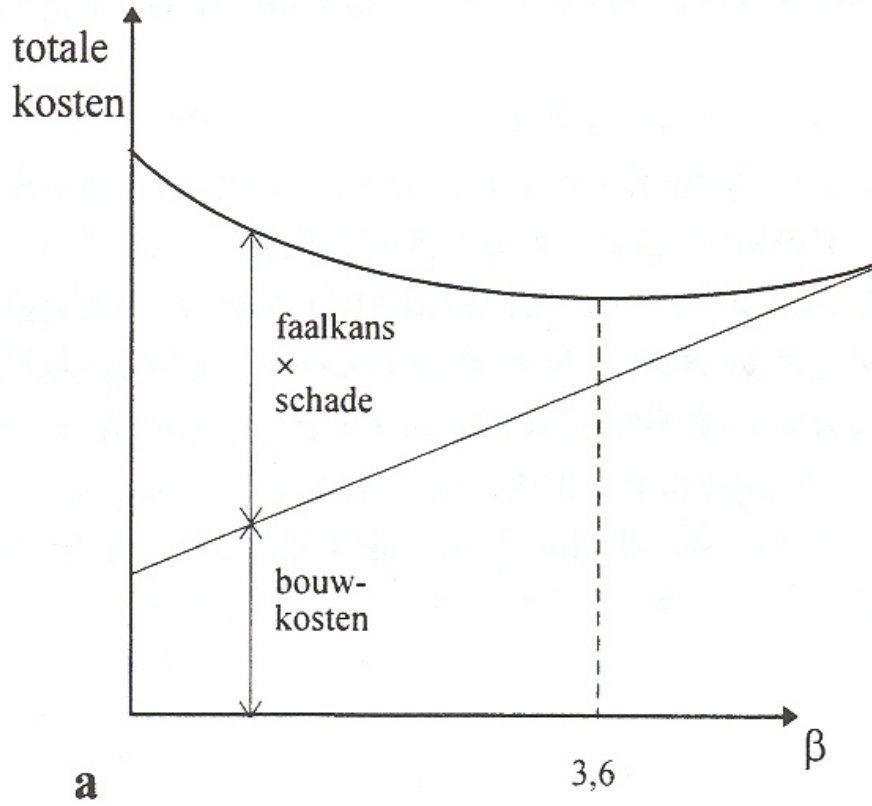
Reliability classes	Consequences for loss of human life, economical, social and environmental consequences	Reliability index $\beta$		Examples of buildings and civil engineering works
		$\beta_a$ for $T_a= 1$ yr	$\beta_d$ for $T_d= 50$ yr	
RC3 – high	High	5,2	4,3	Important bridges, public buildings
RC2 – normal	Medium	4,7	3,8	Residential and office buildings
RC1 – low	Low	4,2	3,3	Agricultural buildings, greenhouses

## Formal Updating formulas

$$f_Q''(q | \hat{x}) = C f_Q'(q) L(|\hat{x}| q)$$

$$f_X^U(x) = \int_{-\infty}^{\infty} f_X(x|q) f_Q''(q | \hat{x}) dq$$

# Cost optimisation / design versus assessment



$$P_F = 10^{-\beta}$$